2 Monge Parametrization

Linear theory - small changes in height, linear in concentration

The equations become

$$\nabla^2 z = H \tag{6}$$

$$C = \mu\phi\sigma \tag{7}$$

$$k\nabla^2(H - C) = p + 2H(\lambda + \beta^2 - 2\alpha\beta\sigma) \tag{8}$$

$$\frac{\partial\lambda}{\partial x} = 2[k\mu\varphi(H - C) + \alpha\beta)]\frac{\partial\sigma}{\partial x} \tag{9}$$

$$\frac{\partial\lambda}{\partial y} = 2[k\mu\varphi(H - C) + \alpha\beta)]\frac{\partial\sigma}{\partial y} \tag{10}$$

$$\frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} = 0 \tag{11}$$

$$\frac{\partial\sigma}{\partial t} + v_1\frac{\partial\sigma}{\partial x} + v_2\frac{\partial\sigma}{\partial y} = 2c\{[\alpha^2 + k(\mu\varphi)^2]\nabla^2\sigma - k\mu\varphi\nabla^2 H\} \tag{12}$$

Parameters: μ, ϕ, c, k, α, β.
Unknowns: S, H, σ, z, v_1 and v_2.
6 equations, 6 unknowns. 3 are second order PDEs, I use the general form PDE, three are first order PDEs.
Domain: a square
Boundary conditions: σ, z and H are known on all four boundaries. S, v_1 and v_2 are provided on one boundary. However, I get a singular matrix when I try to solve this problem. It is a mathematically consistent problem, not over- or under-determined.