Enhancement of Convective Heat Flux through Porous Media, depending on Cell Aspect Ratio

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INTRODUCTION: Convective flow patterns result from the interaction of flow and transport processes, where fluid density is the crucial coupling variable. The formation of convective cells appears in natural and artificial system of various scales: in geology, meteorology, hydraulics, etc.! They are of special interest for heat transfer, i.e. for insulation or cooling of technical devices of many kinds. Here we deal with 2D convection rolls, as they appear near the onset of convection, i.e. near to the critical Rayleigh number (Ra). The heat transfer, described by the dimensionless Nusselt number (Nu) depends on the preferred role pattern, i.e. the number of rolls per unit length ϑ.

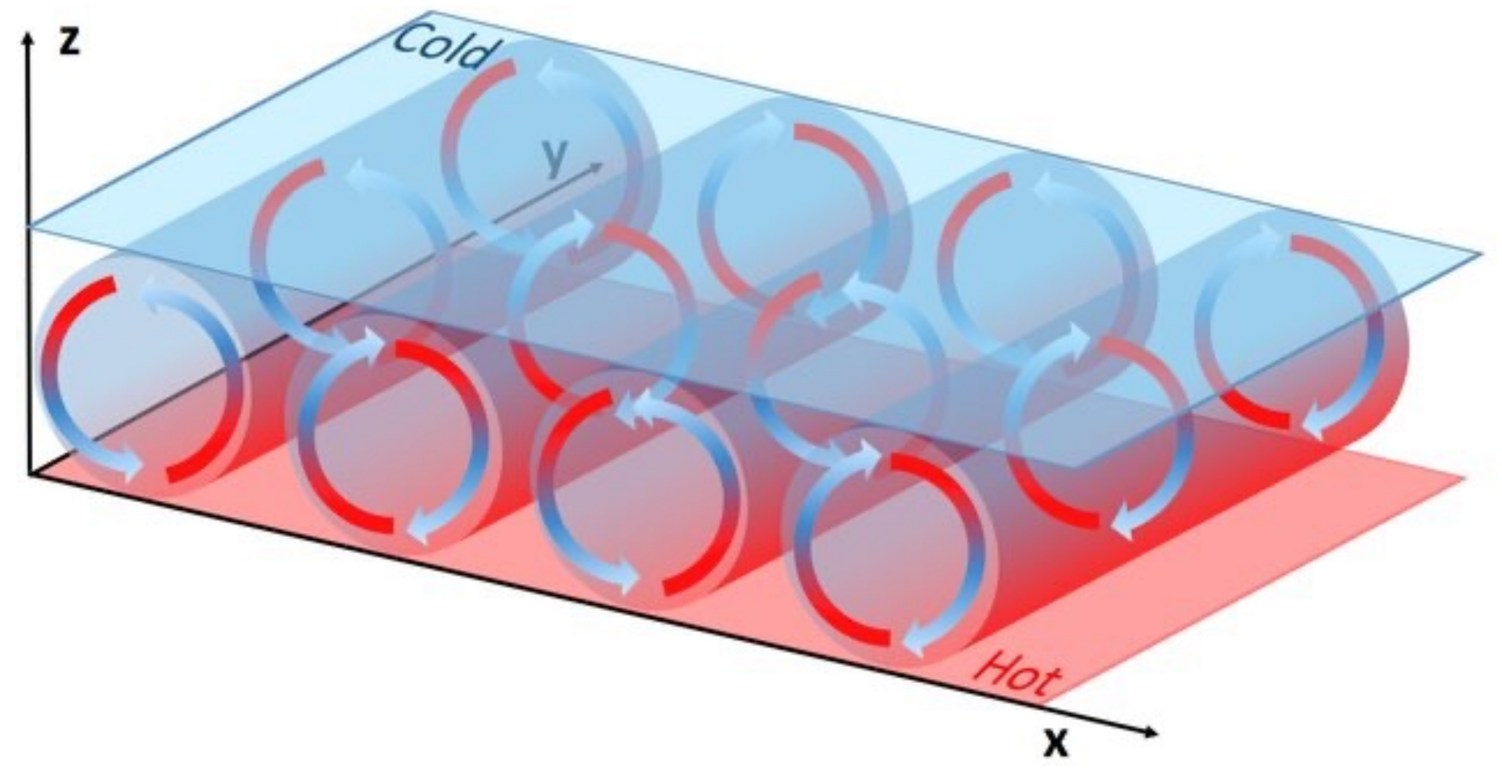

Figure 1. Schematic view of a convection role system (Barna ea. 2017)

The first analytical description of the phenomenon was given by Lord Rayleigh in 1916. The threshold for the onset of convection, i.e. when the temperature difference between the cold plate at the bottom and the hot plate at the top initiates circular flow patterns, is termed in reference the critical Rayleigh number. Here we use the modified parameter combination that is relevant for porous media systems.

DIFFERENTIAL EQUATIONS: Flow and transport in porous media obey the differential equation:

$$\mathbf{v} = -\frac{\mathbf{k}}{\mu}\nabla\left(p-\rho g z\right) \qquad \nabla\cdot\mathbf{v}=0$$

$$\frac{\partial T}{\partial t} = \nabla \mathbf{D}_T \nabla T - \frac{1}{\kappa}\mathbf{v}\nabla T$$

In 2D cartesian (x,z)-coordinates the system can be modified to:

$$\nabla k_x \mathbf{k}^{-1}\nabla\Psi = \mathrm{Ra}\cdot\frac{\partial T}{\partial x}$$

$$\frac{\partial T}{\partial t} = \nabla\frac{1}{D_{Tx}}\mathbf{D}_T\nabla T - \frac{1}{\kappa}\mathbf{v}\nabla T$$

with porous media Rayleigh number $\mathrm{Ra} = \dfrac{k_z g\cdot\Delta\rho\cdot H}{\mu\cdot D_{Tx}}$

Heat transfer is given by the dimensionless Nusselt number:

$$Nu = \frac{1}{L}\int_0^L \frac{\partial\theta}{\partial z}\,dx$$

Anisotropy ratio λ	1	2	3	4	5	10
Critical Rayleigh number Ra_{crit}	$4\pi^2$ 39.48	$4.5\pi^2$ 44.41	$16\pi^2/3$ 52.64	$6.25\pi^2$ 61.69	$7.2\pi^2$ 71.06	$12.1\pi^2$ 119.4

Table 1. Critical Ra for onset of convection

MODEL SET-UP: COMSOL-Multiphysics models are set up using the Poisson-mode for flow in porous media and convection-diffusion mode for heat transport. We use cubic shape functions. The mesh consists of 48128 triangular elements, and 435074 dof. The presented results are an update of a diagram, obtained by a Finite Difference code simulation on a coarse mesh (Holzbecher 1996).

RESULTS: The numerical solution visualizes the convection cells. Their shape and number of cells per length depend on the Ra-number.

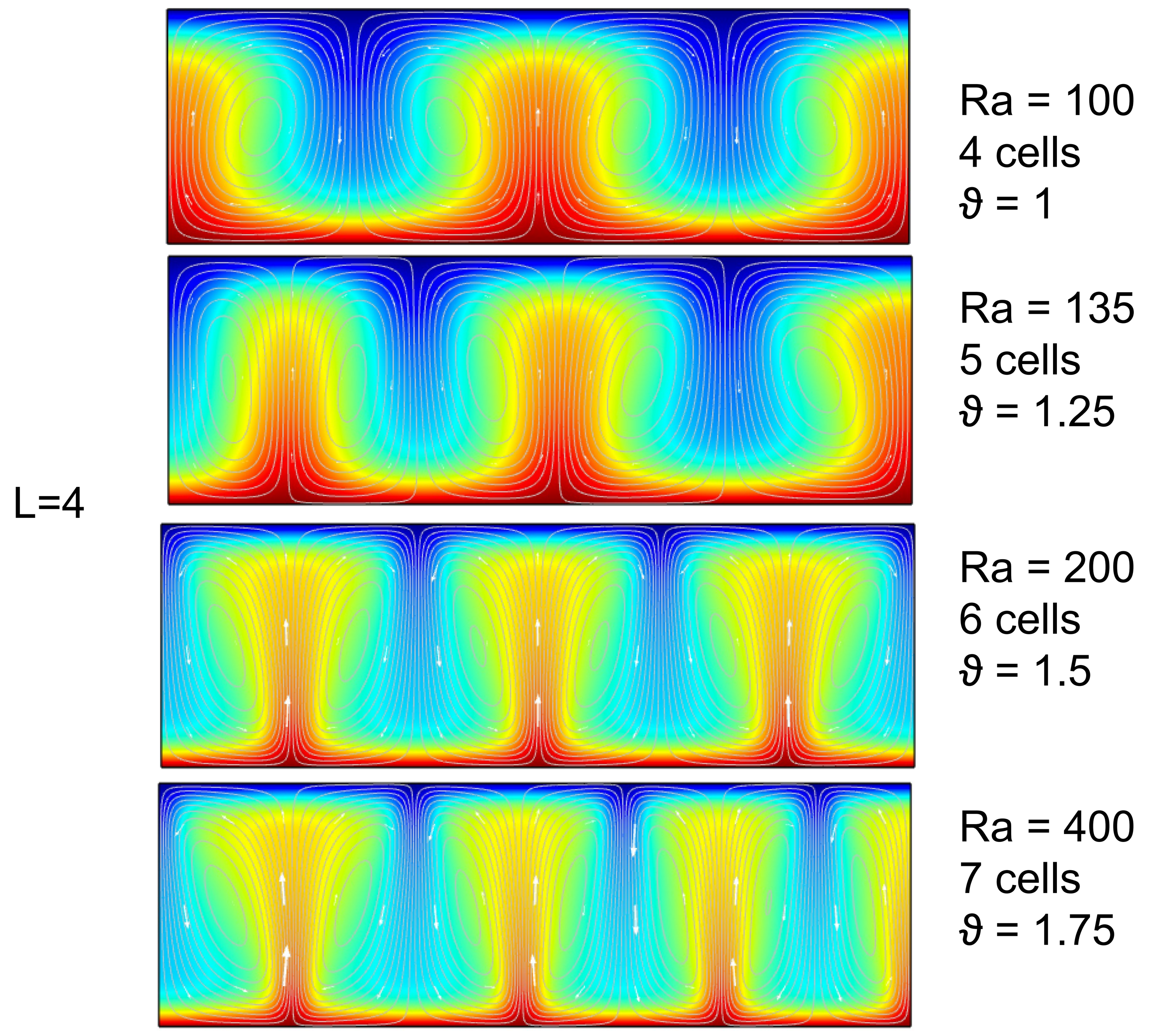

Figure 2. Preferred cell modes, in dependence of Ra

The heat transfer through the system depends on the convective flow pattern. Figure 3 shows the Nu-number that represents the total heat transfer as function of cells per unit and Ra-number.

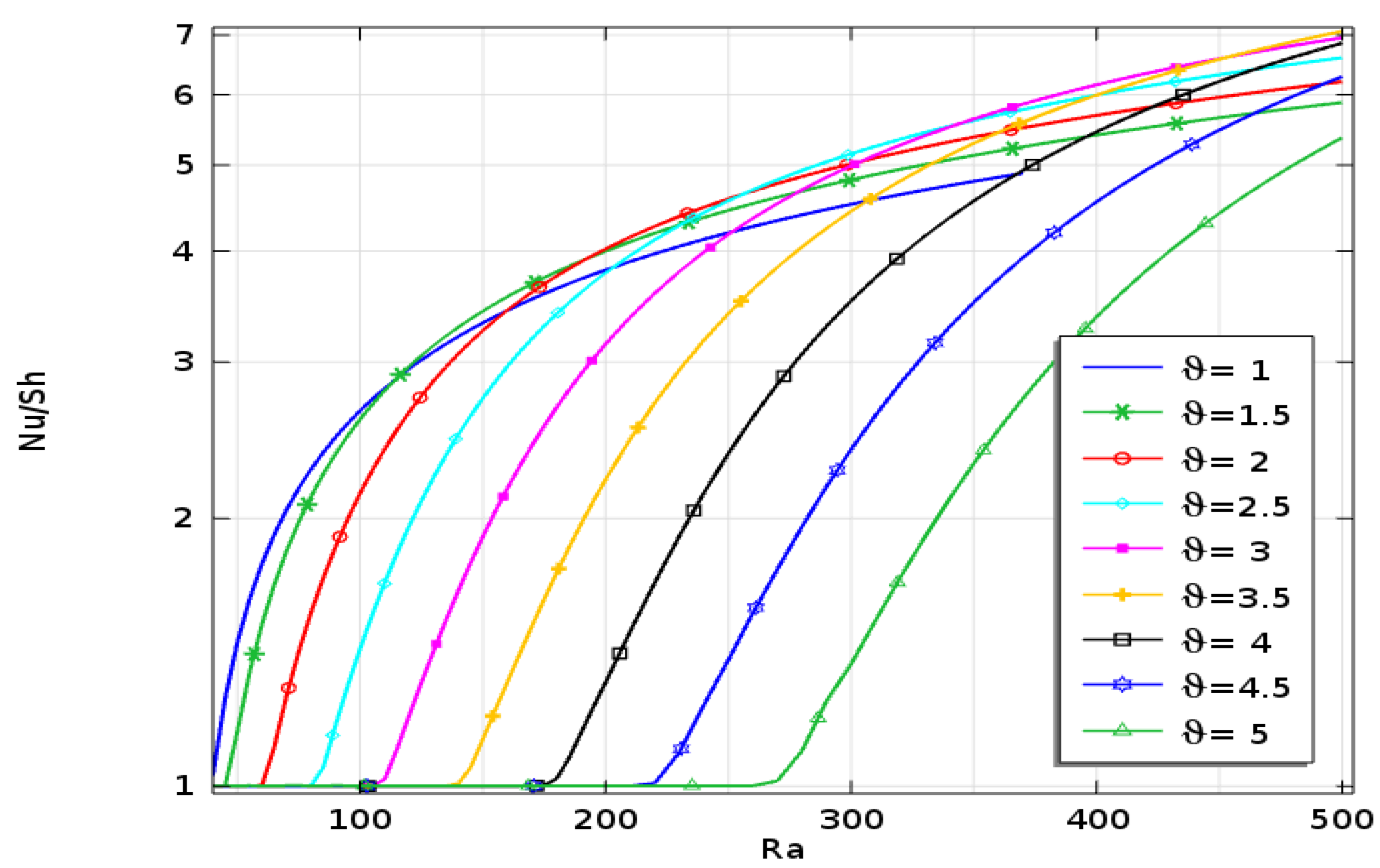

Figure 3. Ra-Nu diagram depending on Ra number and aspect ratio

CONCLUSIONS: The knowledge of the quantitative dependency of heat transfer on the convection cell size can be utilized in the design of insulating or cooling devices. Minor technical modifications on the horizontal walls can favour the emergence of a preferred mode, i.e. the cell pattern with lowest heat transfer for thermal insulation and the pattern with highest heat transfer for cooling.

REFERENCES:

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