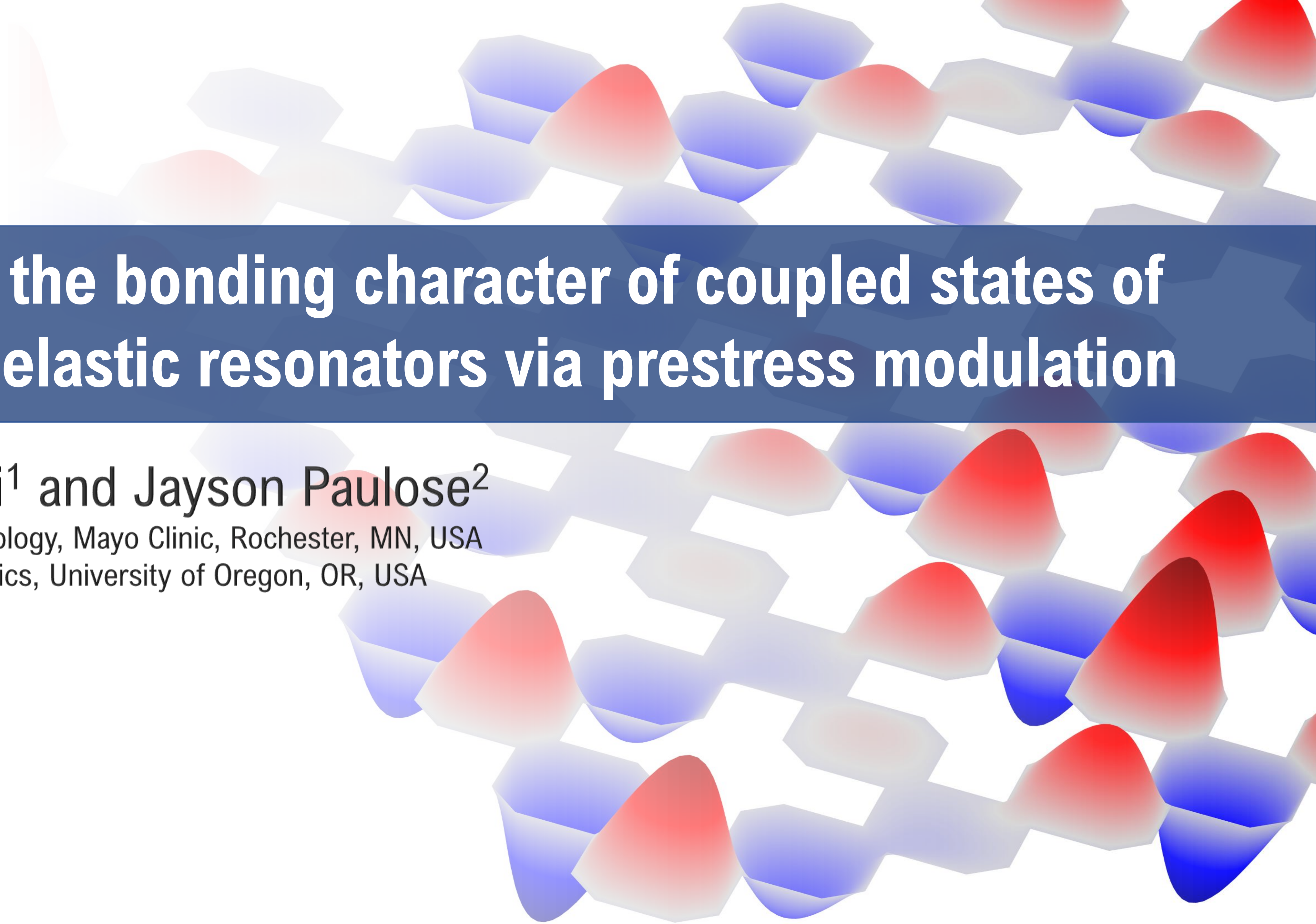
Modifying the bonding character of coupled states of thin-plate elastic resonators via prestress modulation

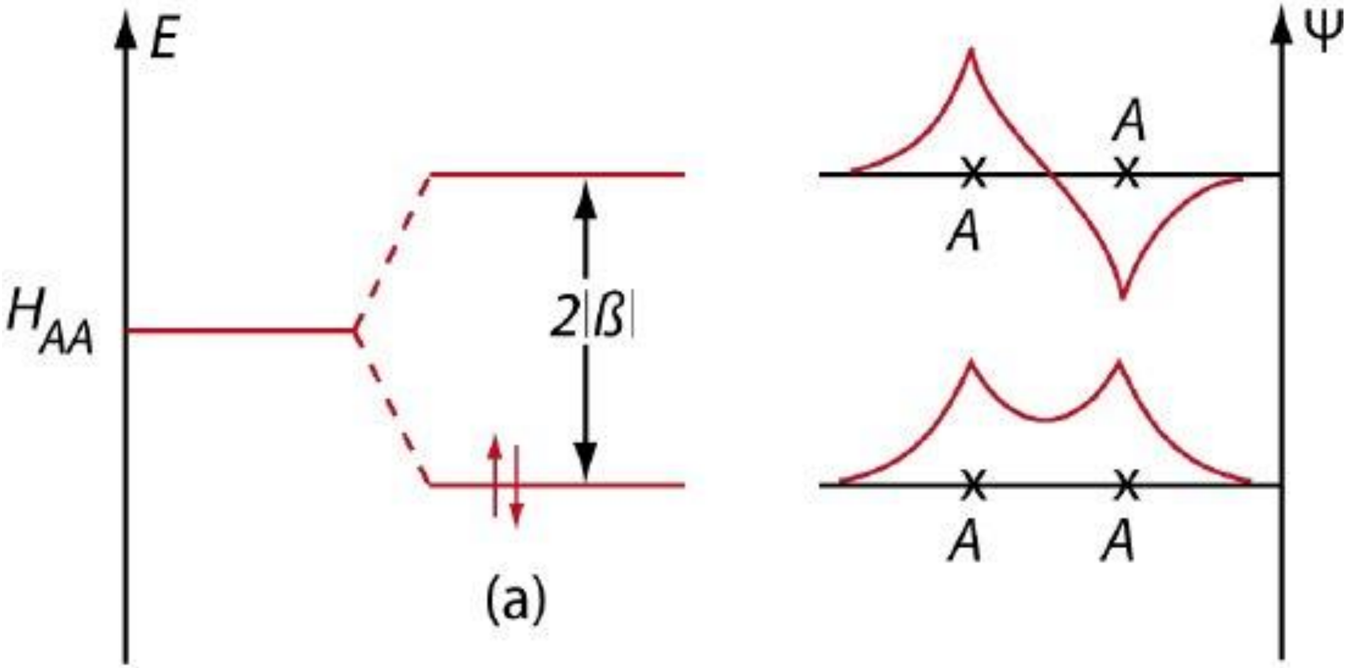
Pragalv Karki[1] and Jayson Paulose[2]

1. Department of Radiology, Mayo Clinic, Rochester, MN, USA
2. Department of Physics, University of Oregon, OR, USA



Bonding character

Diatomic molecule with expected bonding character



D. G. Pettifor, arXiv:1112.4638

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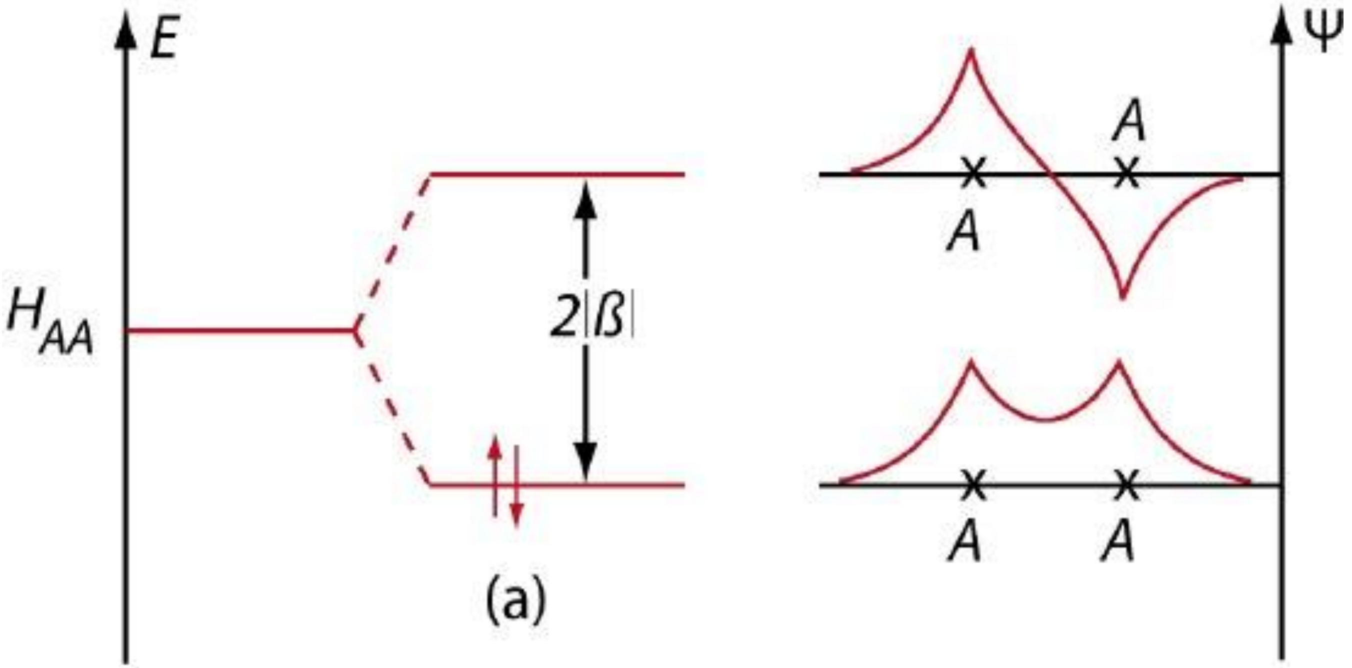

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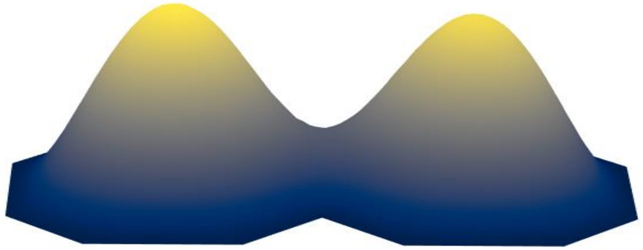
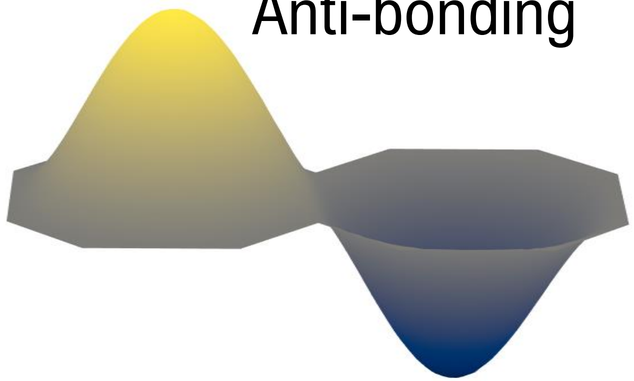
Reversing the bonding character in thin elastic plates

Bonding



Anti-bonding

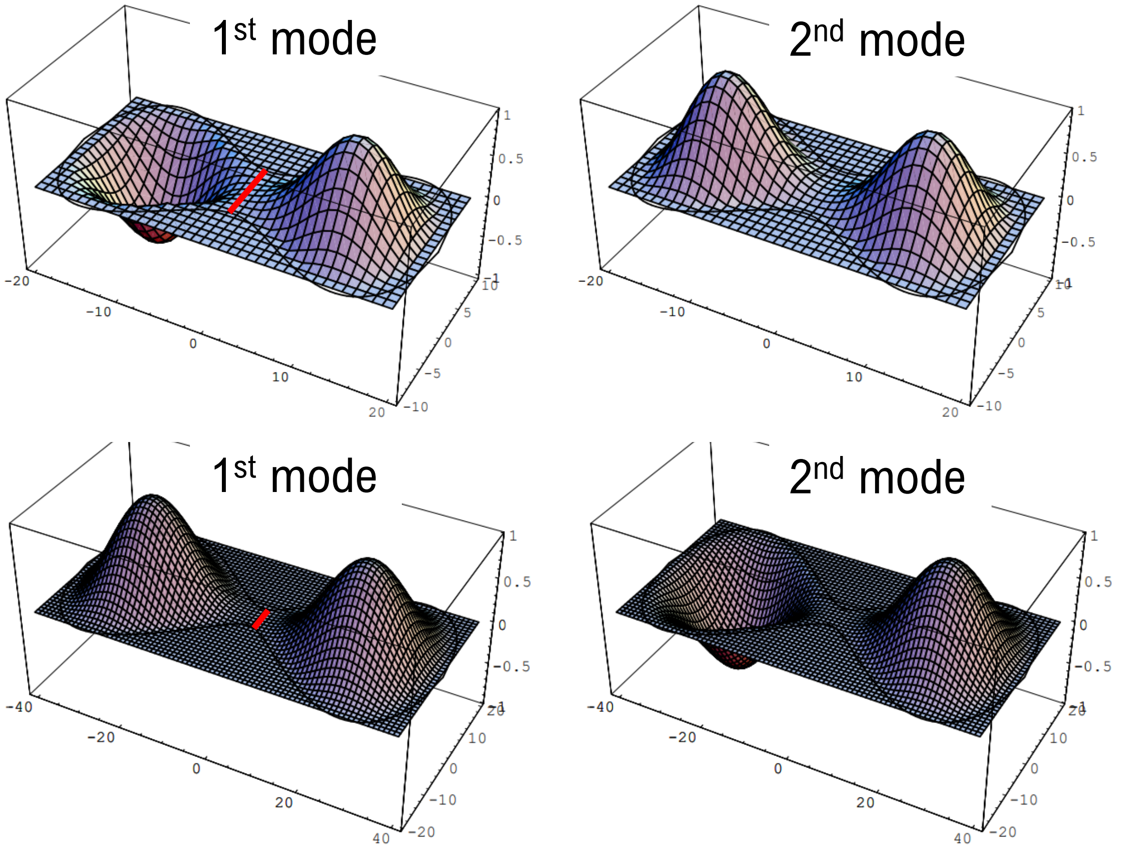


Fourth order plate to thin elastic plate

Fourth order plate

$$\nabla^4 u = 0 \quad \text{on domain}$$

$$u = \nabla u = 0 \quad \text{clamped on boundary}$$



G. Sweers, *Electronic Journal of Differential Equations (EJDE)*, 2001

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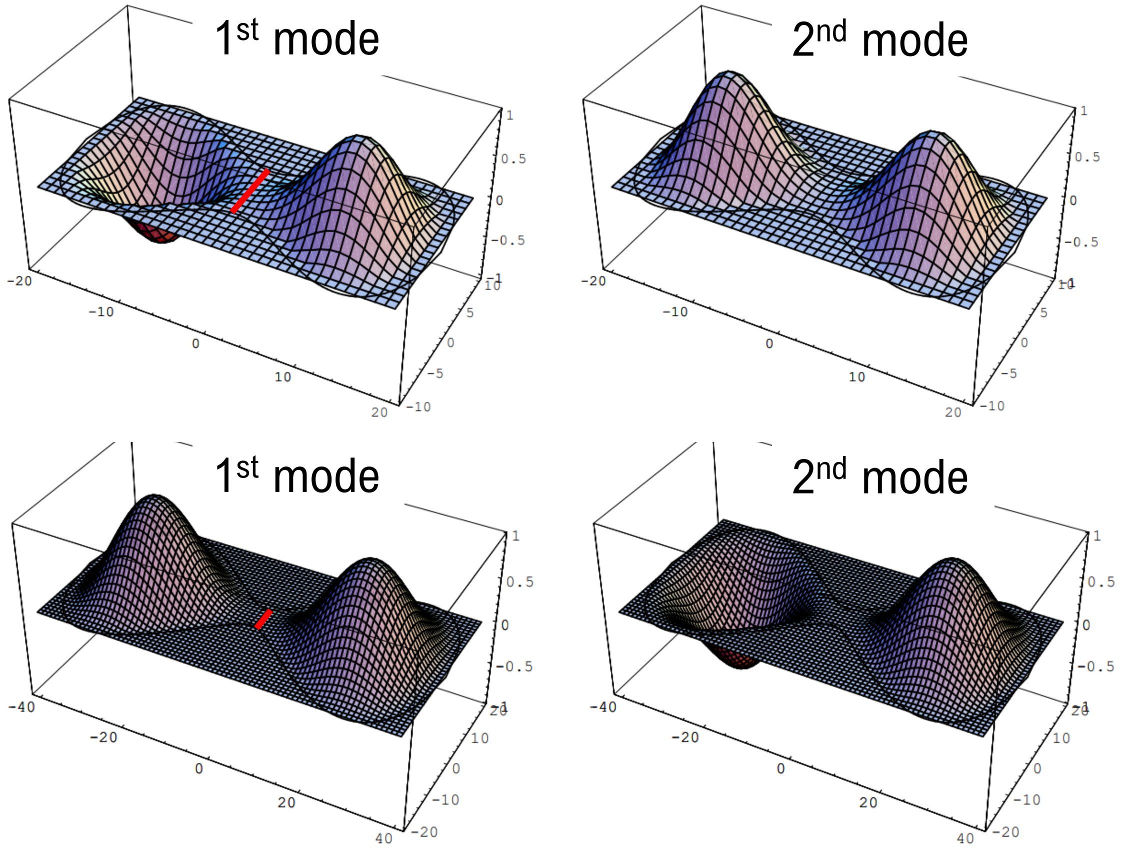

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Thin elastic plate (Föppl–von Kármán)

$$\nabla^4 u - T\nabla^2 u = 0 \quad \text{on domain}$$

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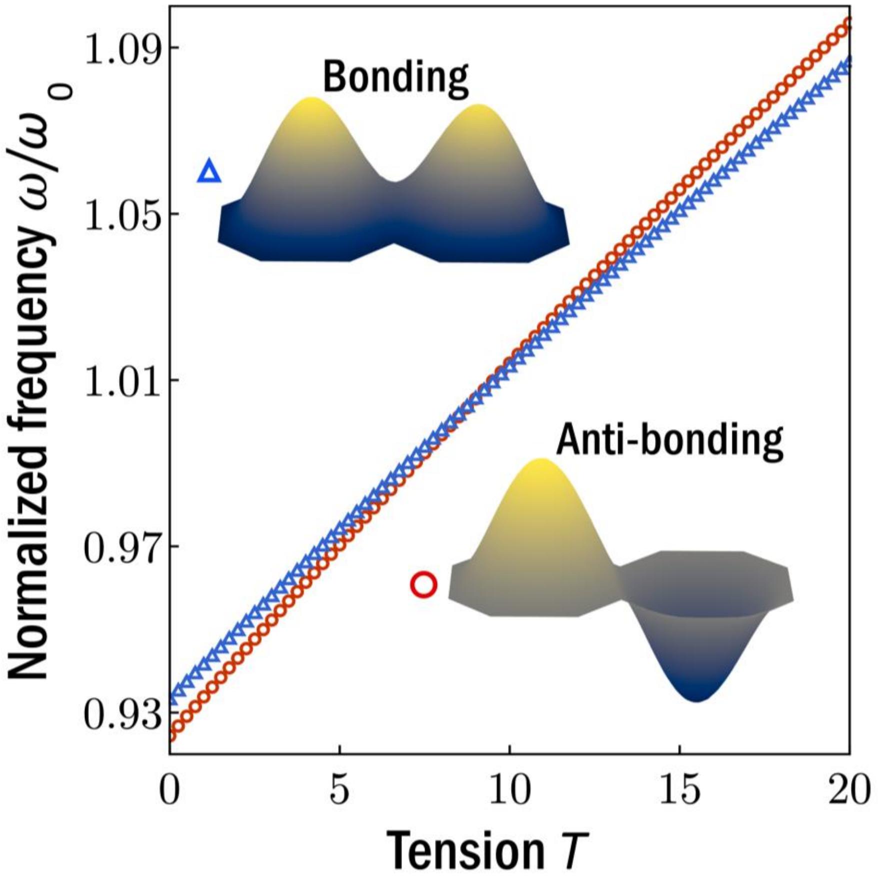

Thin elastic plate (Föppl–von Kármán)

Full dynamical equation

$$\rho \frac{\partial^2 u}{\partial t^2} + c \frac{\partial u}{\partial t} + D\nabla^4 u - T'\nabla^2 = 0$$

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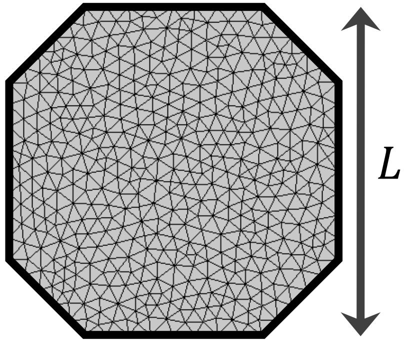

Rescale

$$\bar{x} = x/L, \bar{y} = y/L, \bar{t} = t\sqrt{D/(\rho L^4)}$$

Non-dimensional form

$$\frac{\partial^2 u}{\partial \bar{t}^2} + \zeta\frac{\partial u}{\partial \bar{t}} + \bar{\nabla}^4 u - T\bar{\nabla}^2 u = 0$$

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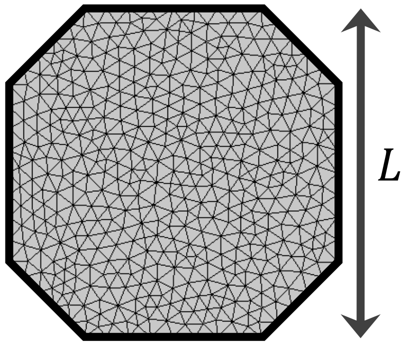

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Drop the bars

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$$T \equiv \frac{T'L^2}{D}$$

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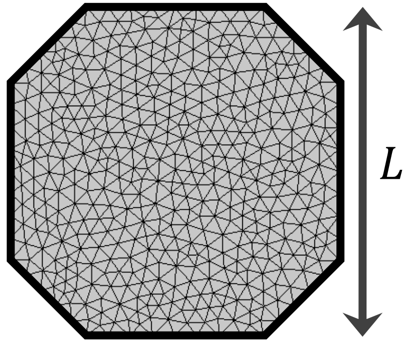

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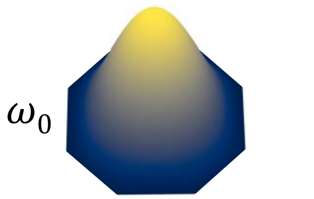
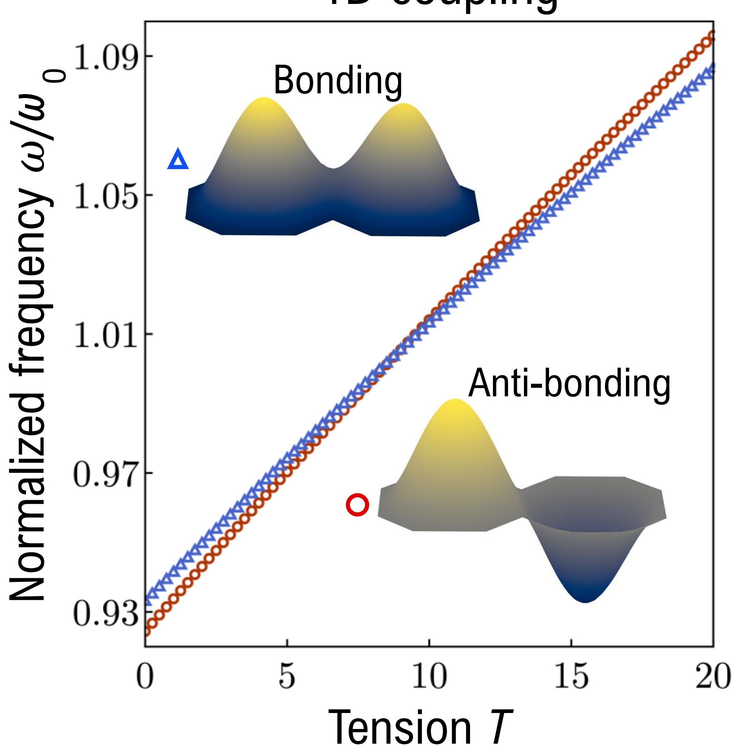
Eigenvalue problem

$$\nabla^4 u-T\nabla^2 u=\omega^2 u$$

Normalizing mode

T = 0





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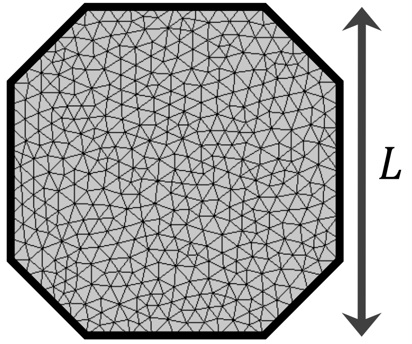

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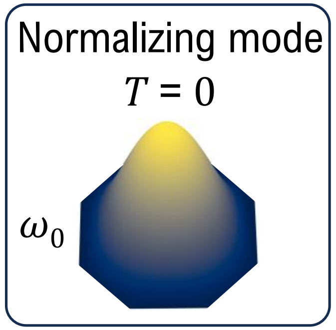

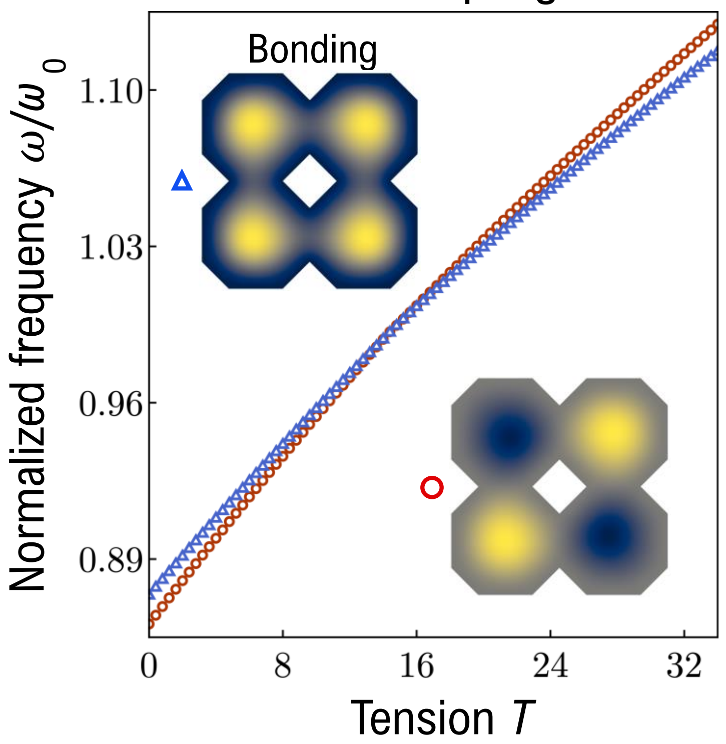

COMSOL implementation

Eigenvalue problem

$$\nabla^4 u - T\nabla^2 u = \omega^2 u$$

Eigenvalue Study in General Form PDE

$$\lambda^2 e_a \mathbf{u} - \lambda d_a \mathbf{u} + \nabla.\Gamma = f$$

$$\mathbf{u} = [u1, u2, u3, u4, u5]^T$$

$$\nabla = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right]$$

$$\Gamma_1 = \begin{bmatrix} u4_x + 2u5_x - Tu1_x \\ u1 \\ 0 \\ u2 \\ 0 \end{bmatrix} \quad \Gamma_2 = \begin{bmatrix} u5_y - Tu1_y \\ 0 \\ u1 \\ 0 \\ u3 \end{bmatrix} \quad f = \begin{bmatrix} 0 \\ u2 \\ u3 \\ u4 \\ u5 \end{bmatrix}$$

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We get 5 sets of equations

I) $(u4_{xx} + 2u5_{xx} + u5_{yy}) - T(u1_{xx} + u1_{yy}) = \lambda u1$

II) $u2 = u1_x$

III) $u3 = u1_y$

IV) $u4 = u2_x$

V) $u5 = u3_y$

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II) $u2 = u1_x$

III) $u3 = u1_y$

IV) $u4 = u2_x$

V) $u5 = u3_y$

Which simplifies to

$$\left(\frac{\partial^4 u1}{\partial x^4} + 2\frac{\partial^4 u1}{\partial x^2 \partial y^2} + \frac{\partial^4 u1}{\partial y^4}\right) - T\left(\frac{\partial^2 u1}{\partial x^2} + \frac{\partial^2 u1}{\partial y^2}\right) = \lambda u1,$$

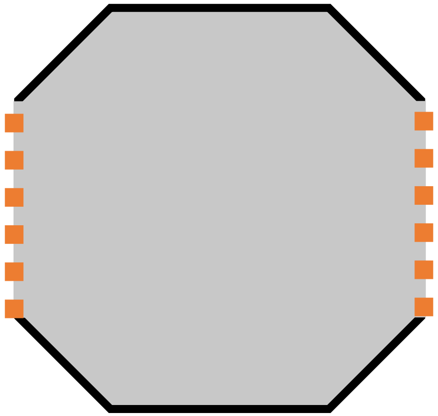
$$\nabla^4 u1 - T\nabla^2 u1 = \lambda u1$$

Band structure calculation

Bloch periodicity:

$$u_{k_x}(x,y) = e^{ik_x x}\phi_{k_x}(x,y)$$

1D lattice



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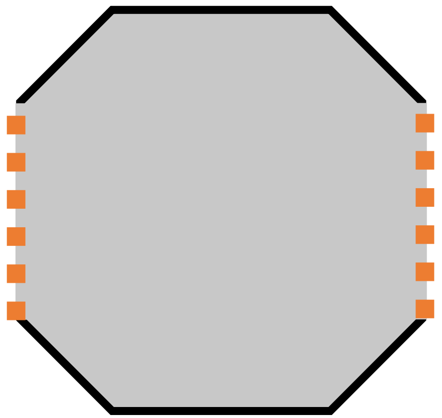

Bloch periodicity:

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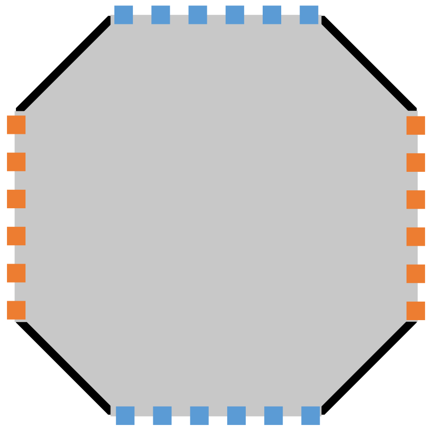
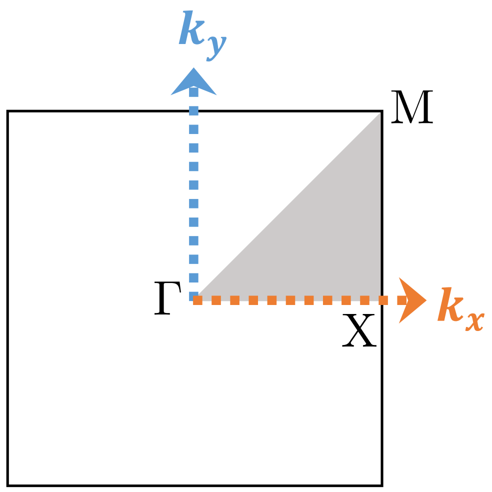
2D square lattice



1st Brillouin zone



Band structure calculation – 1D

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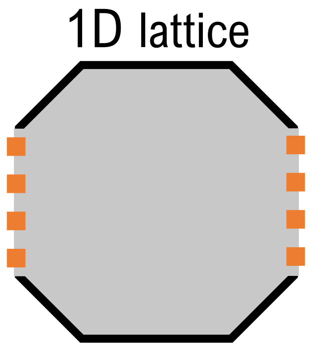
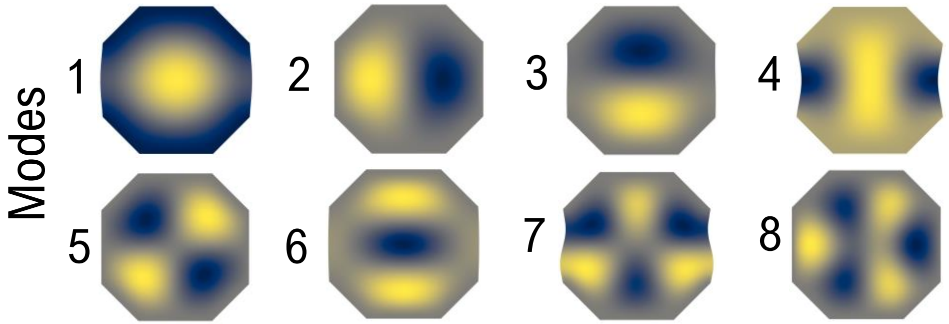
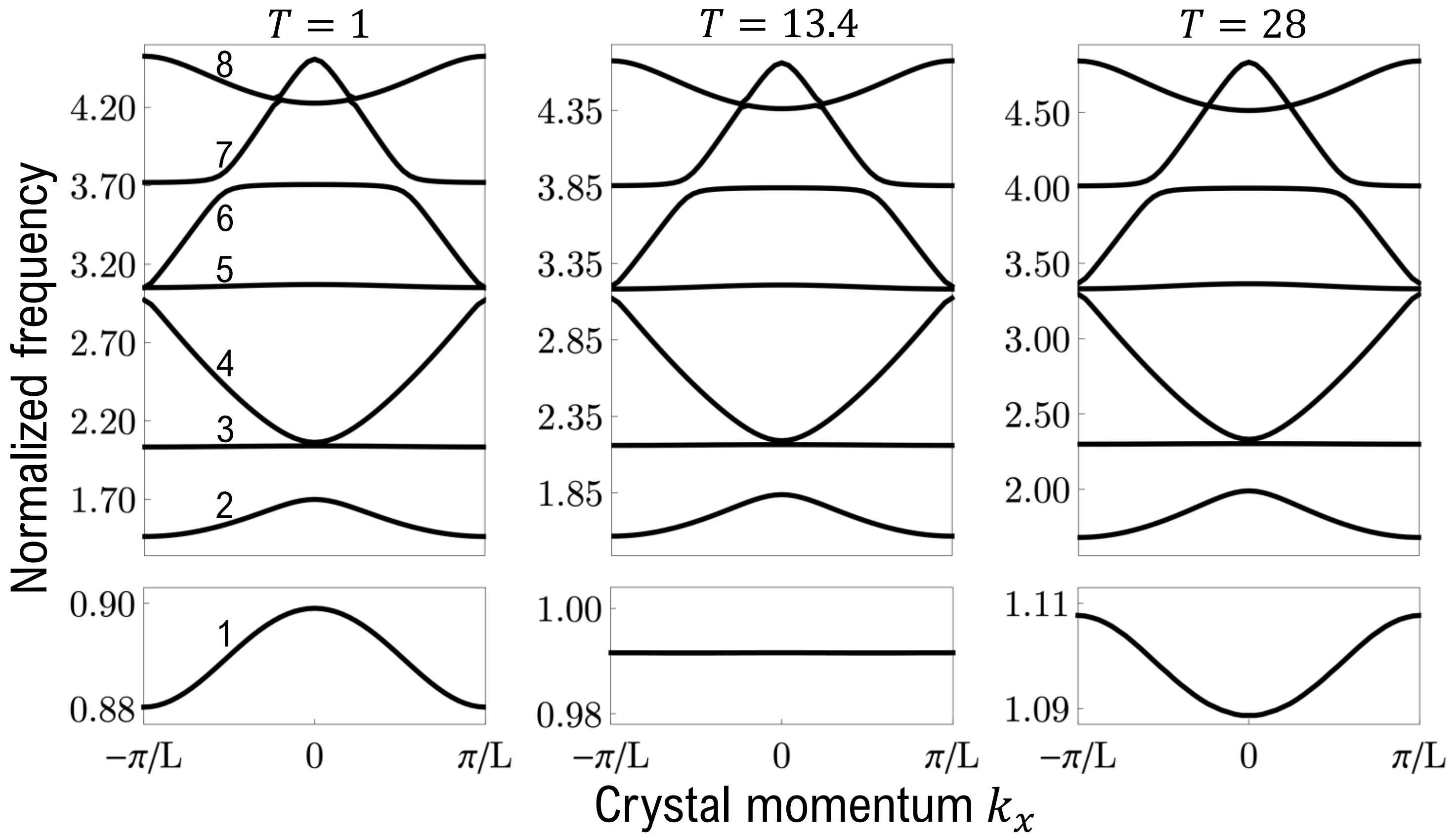
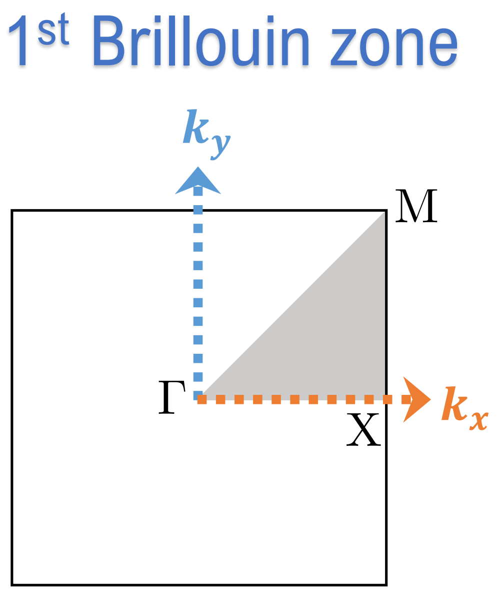
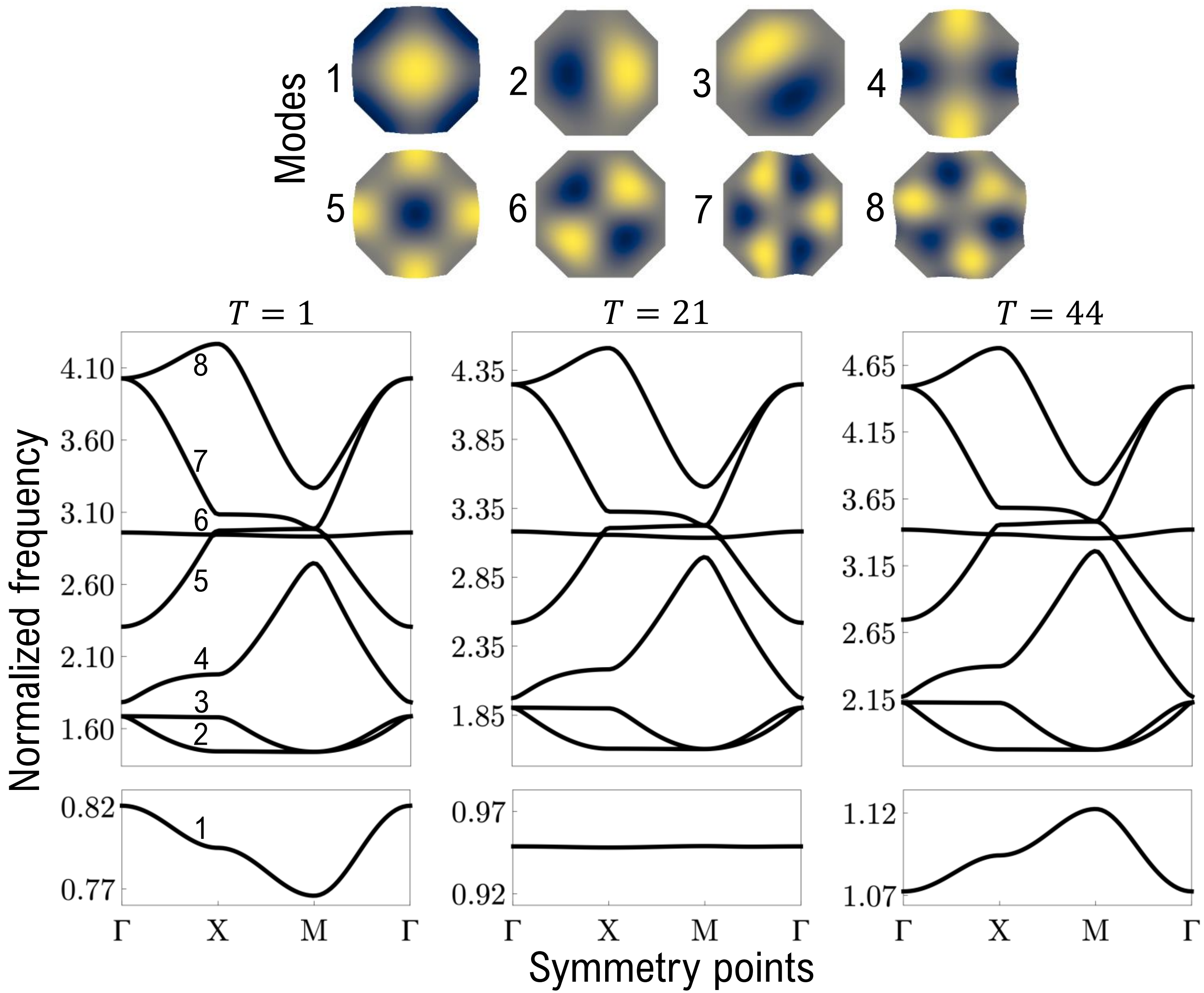






Band structure calculation – 2D

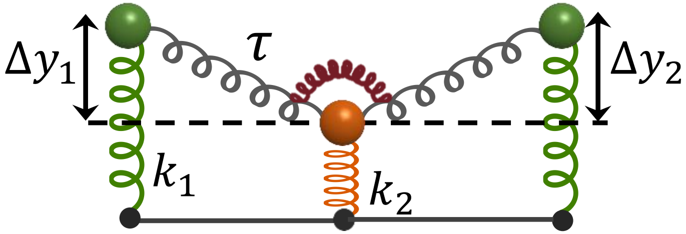




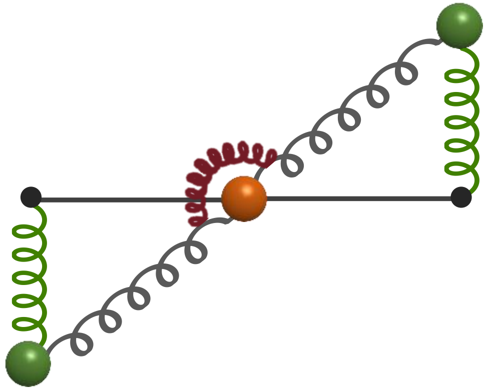
Mapping to a discrete spring mass model

P. Karki and J. Paulose, *Physical Review Applied*, 2021.

Bonding



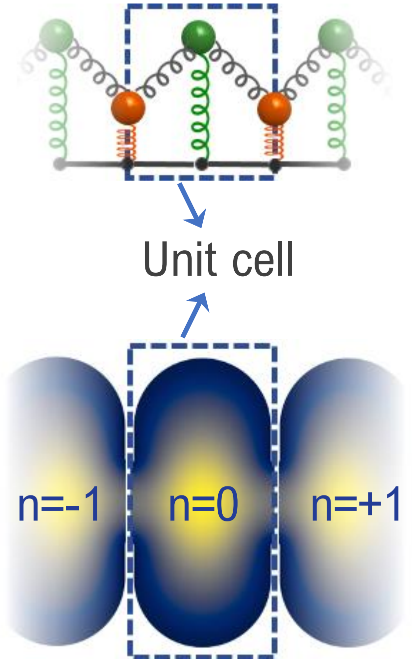
Anti-bonding



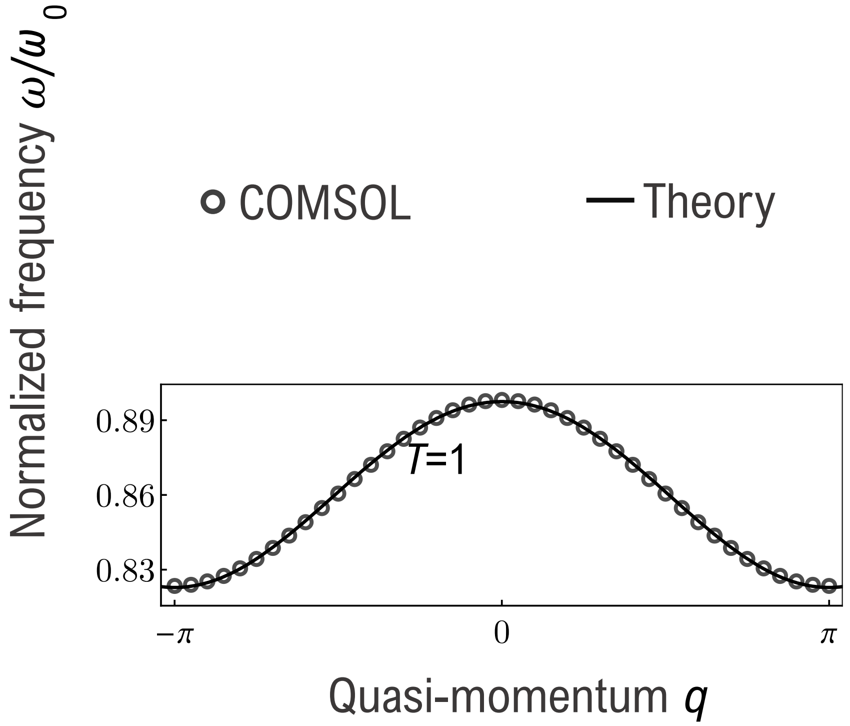
Band structure of 1D chain

Periodic boundary condition

$$\mathbf{u}(n,t) = \mathbf{u}_0 e^{i(qn-\omega t)}$$



$$u_q(x,y) = \phi_q(x,y)e^{iqn}$$



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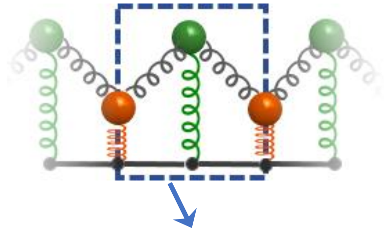
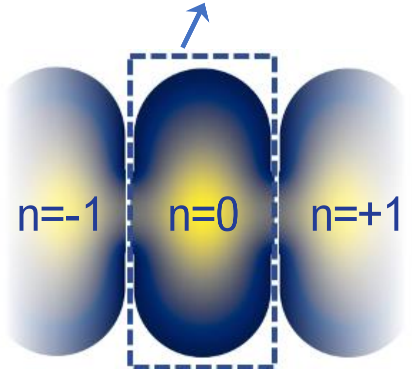


Unit cell



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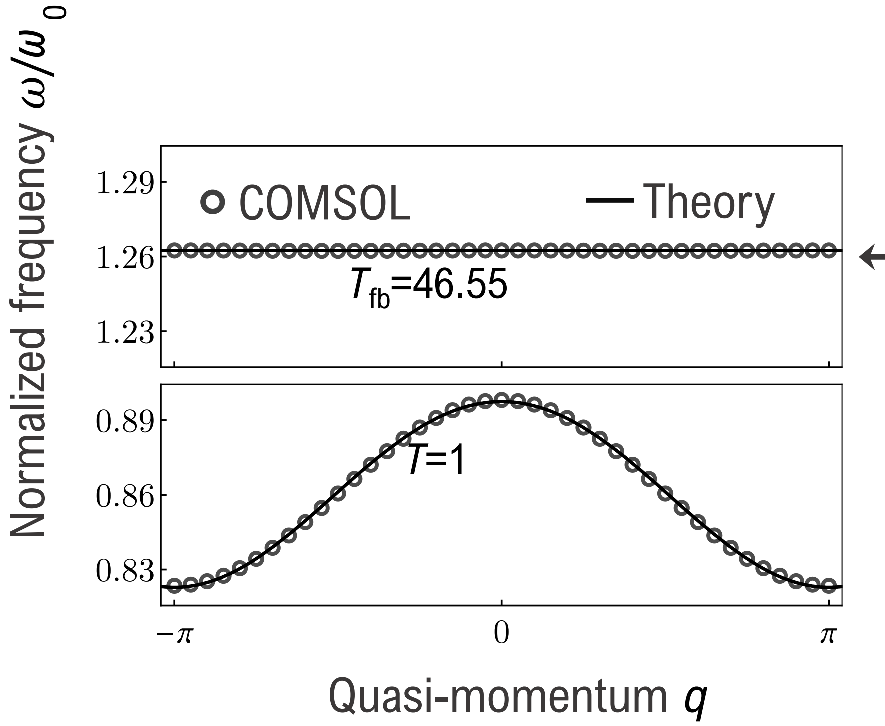
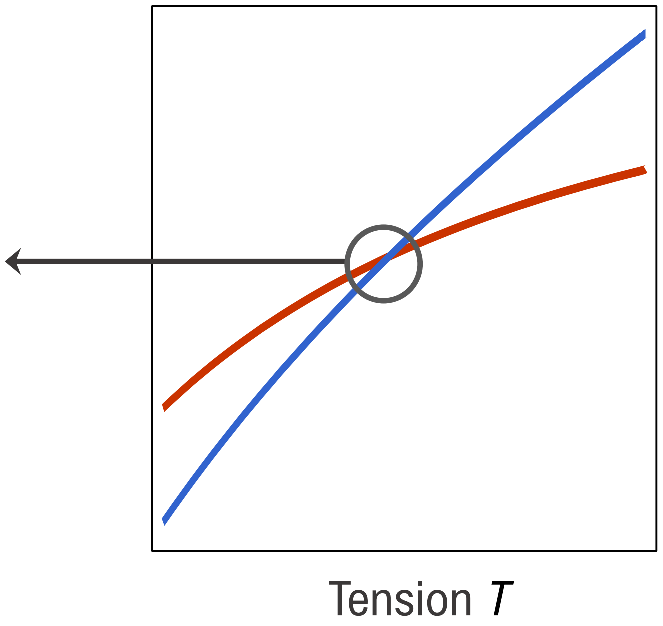


Mode-crossing

ω vs T



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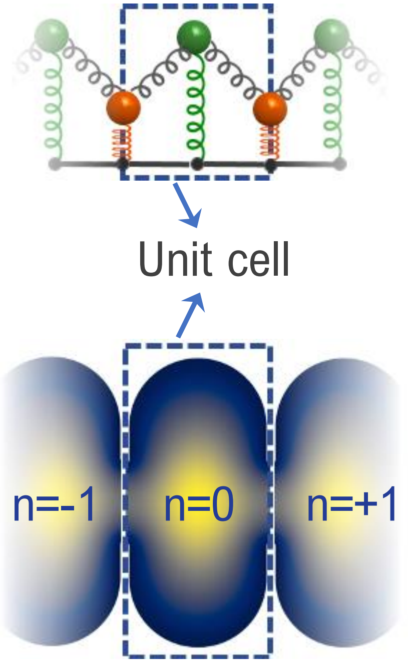

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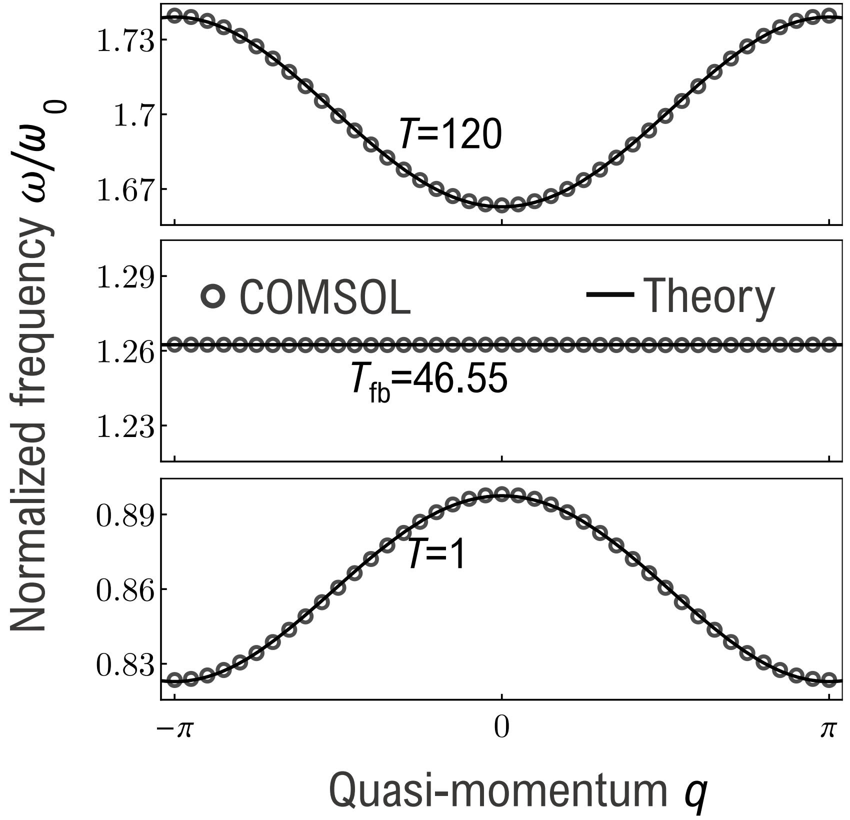

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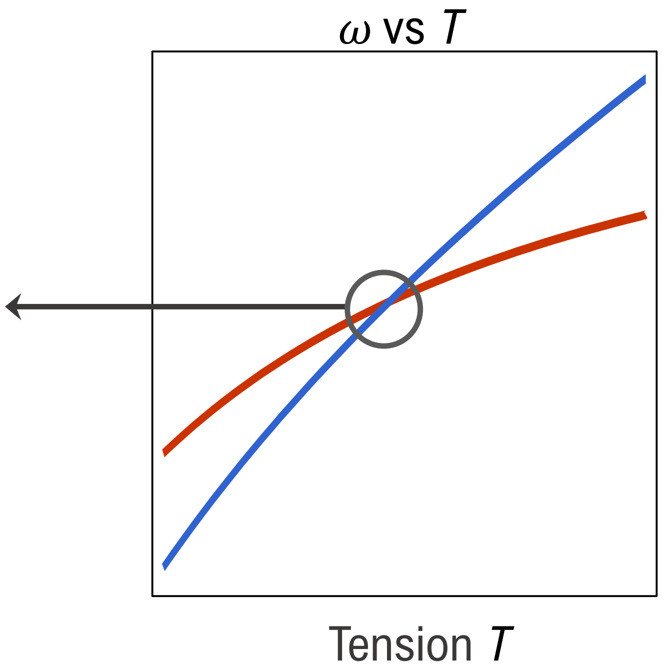
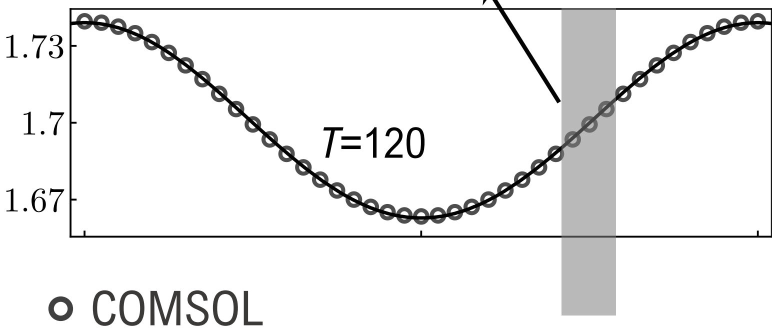
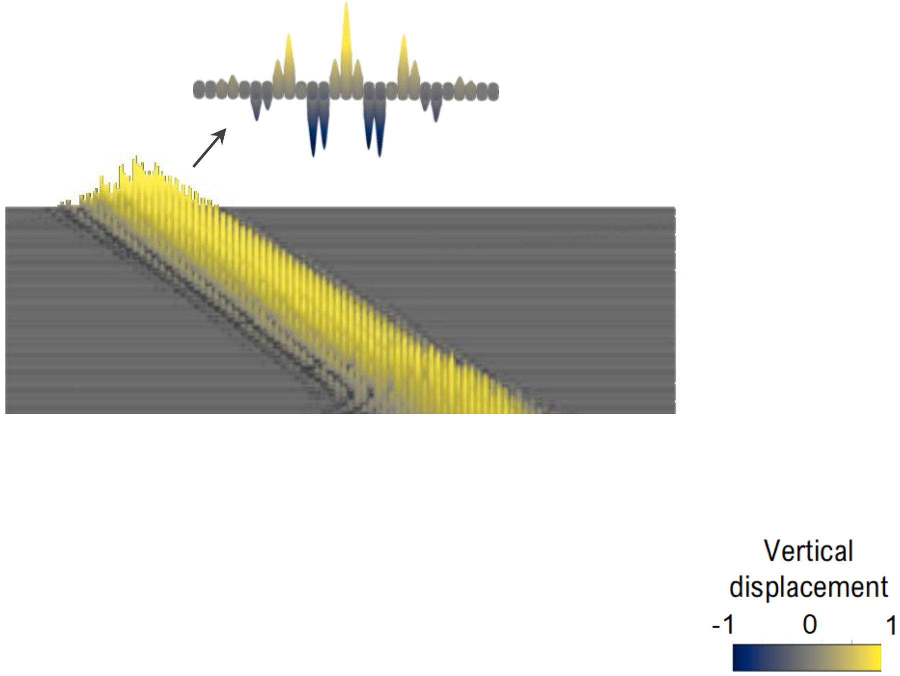


Stopping and reversing sound

Gaussian wave-packet $\boldsymbol{u}(n,t) = \sum \phi_{q_x} e^{i(q_x n - \omega(q_x)t)} e^{-\left(\frac{q_x - q_0}{\Delta q_x}\right)^2}$





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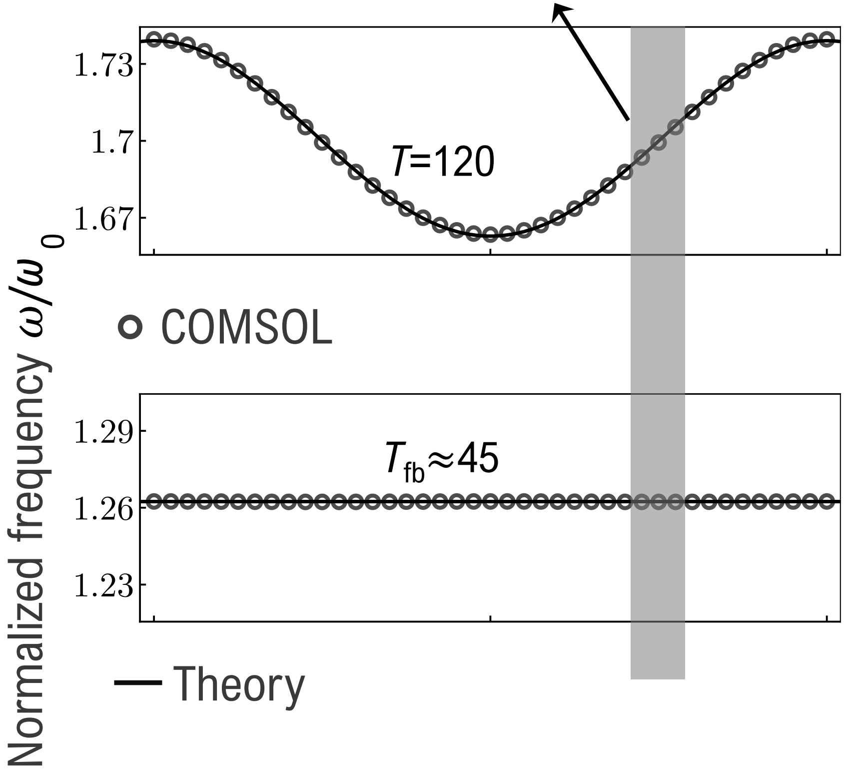

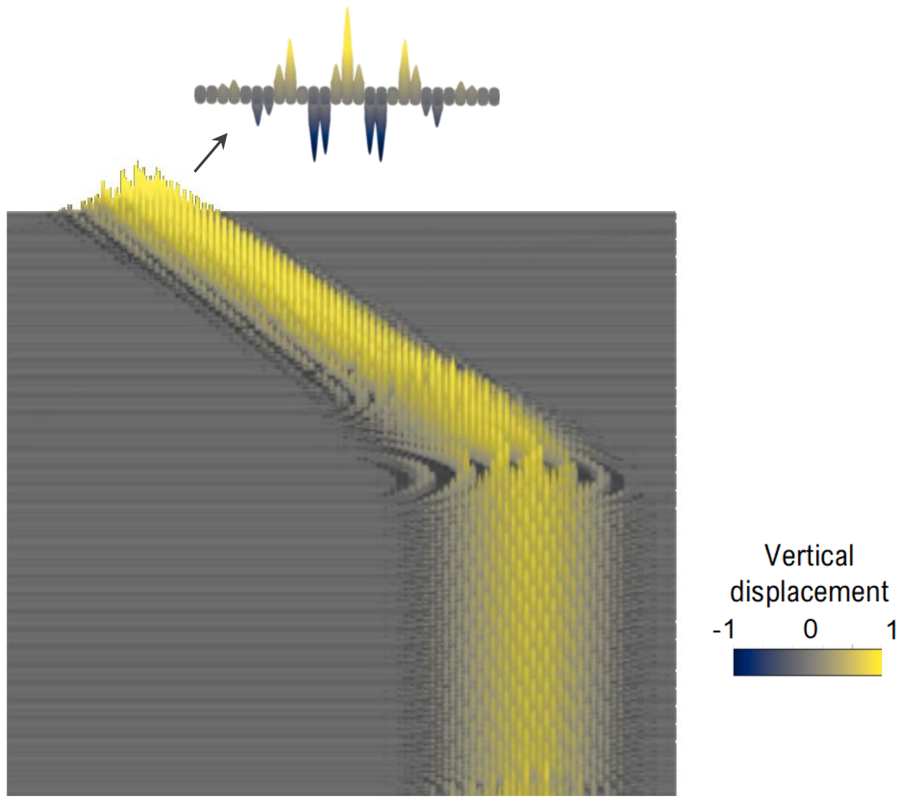

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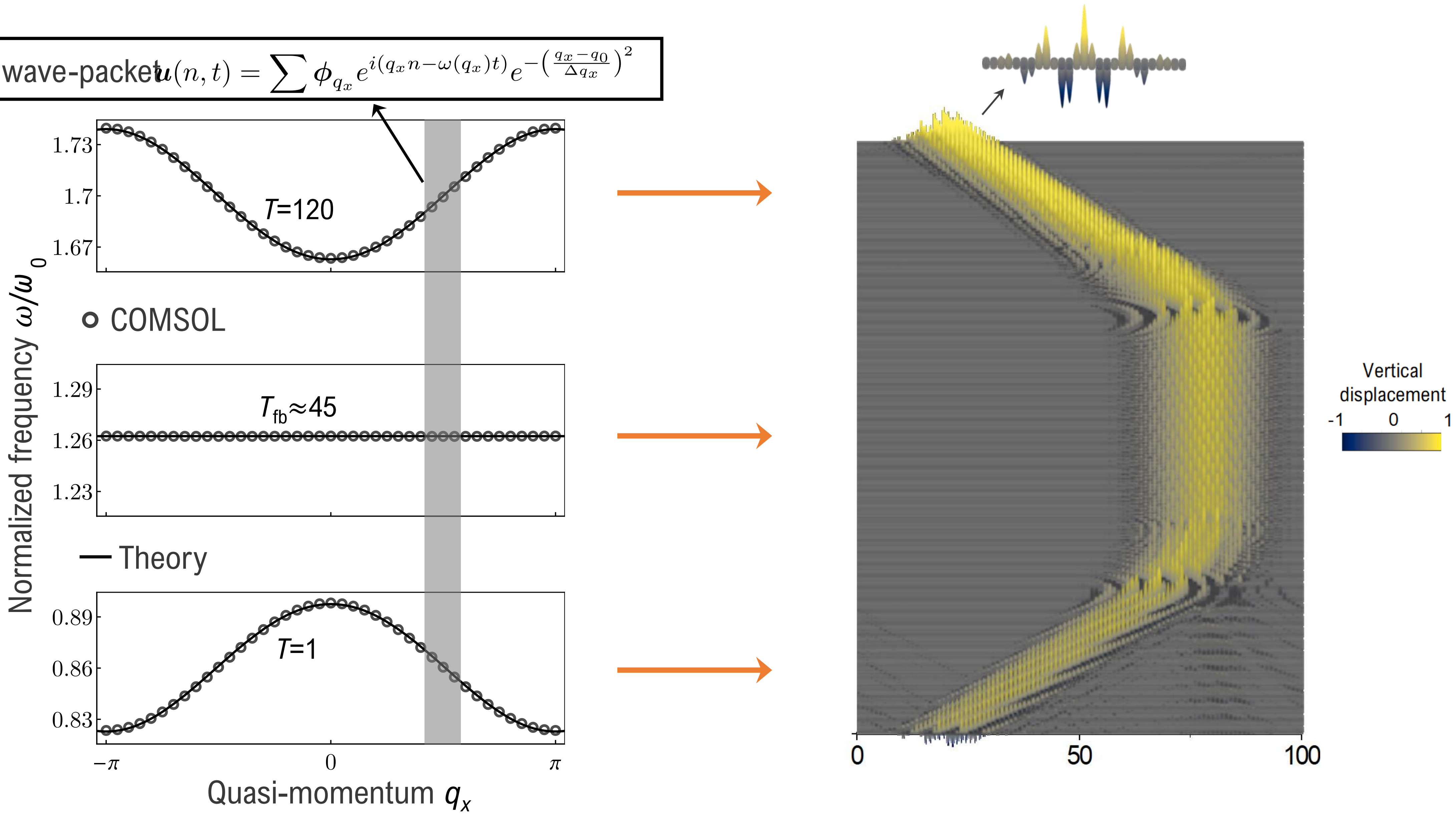

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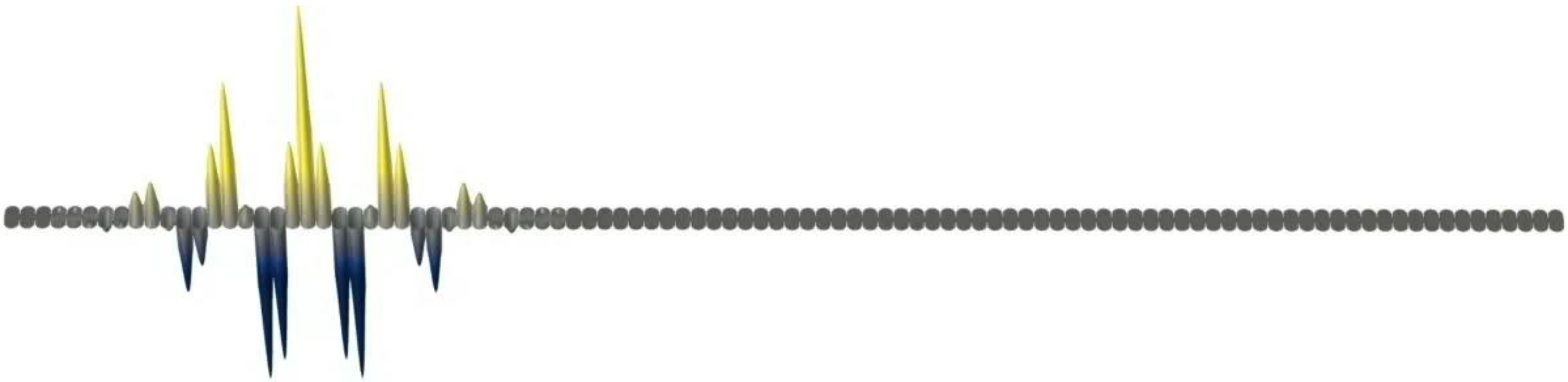

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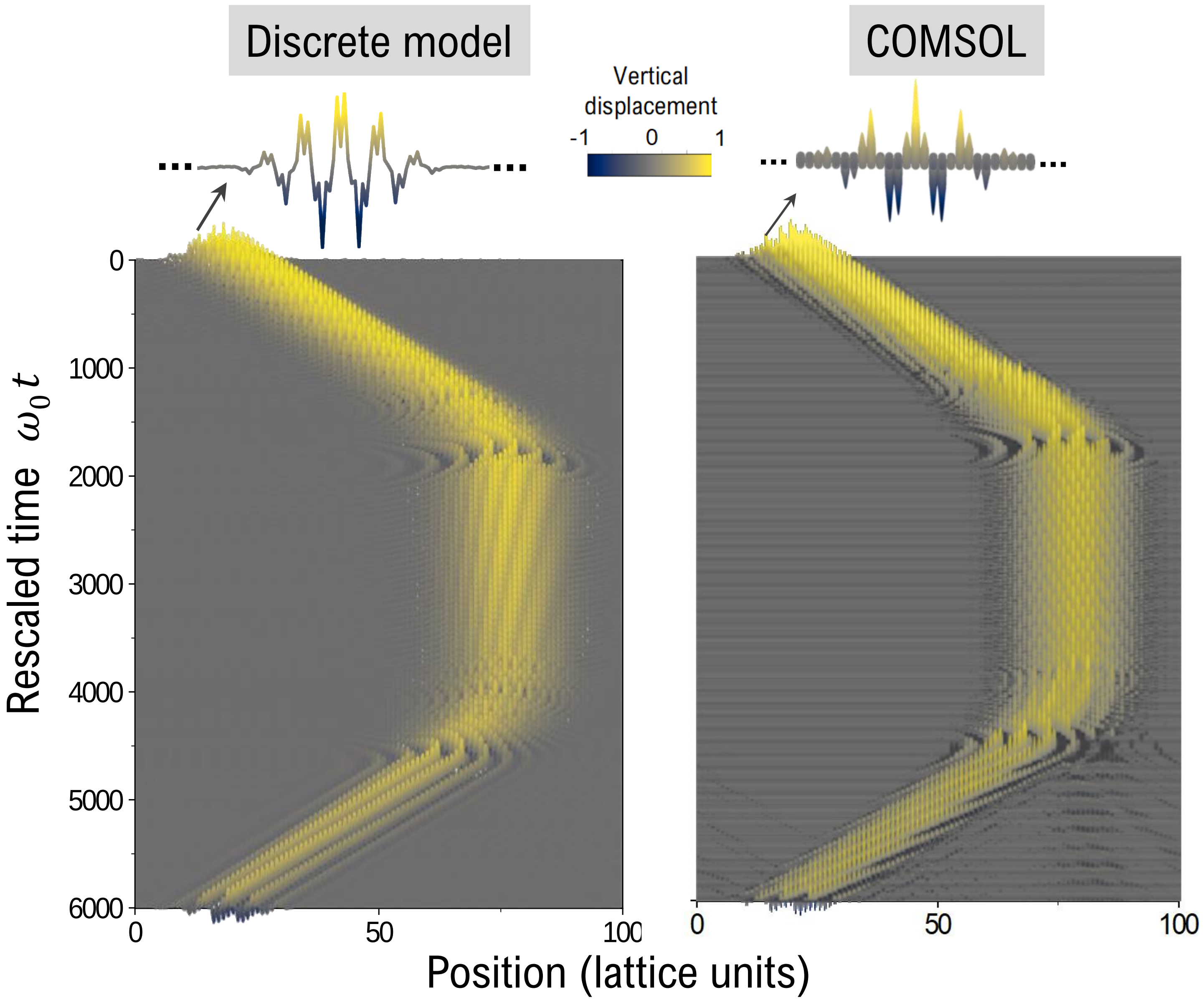

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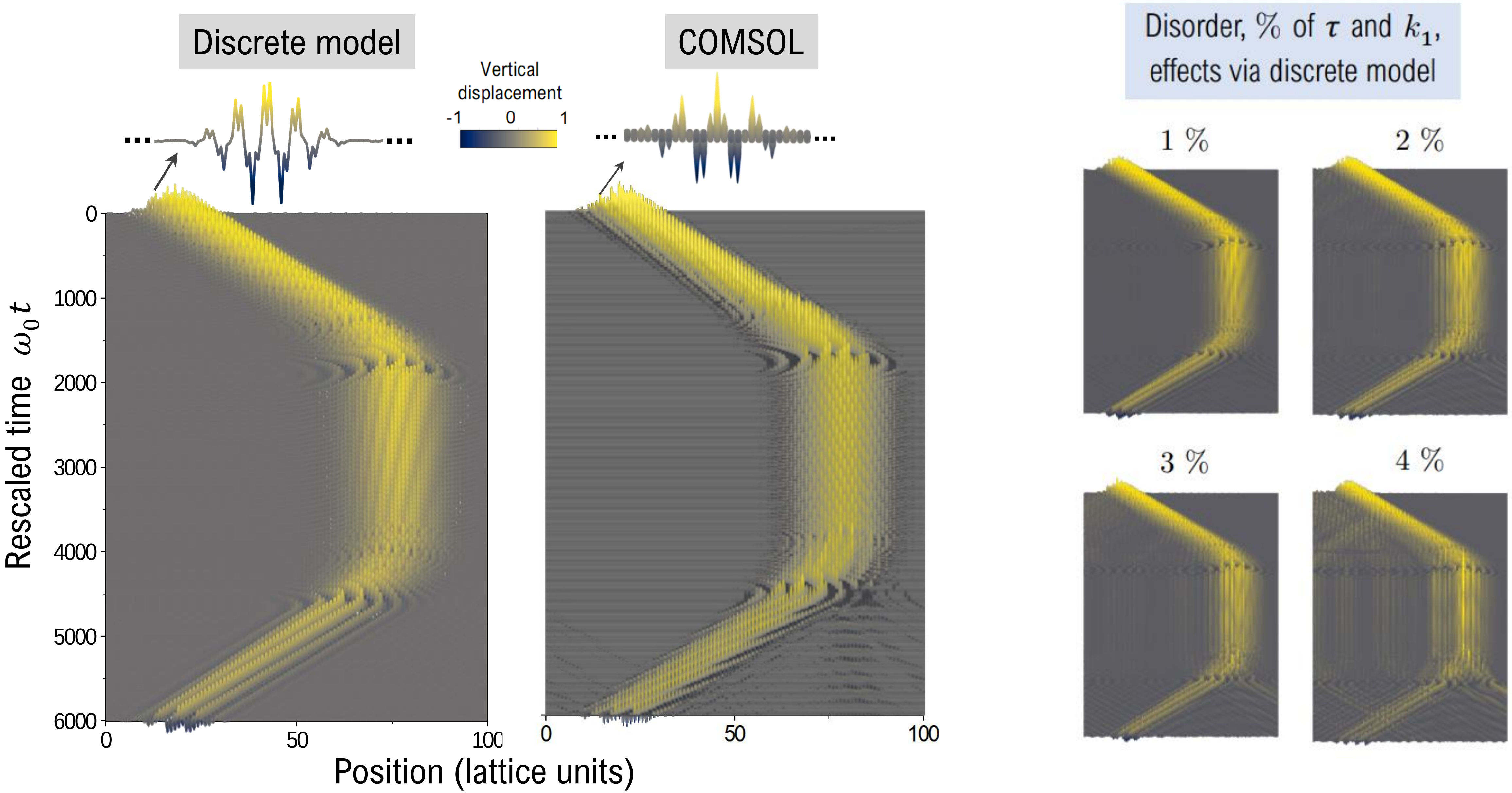
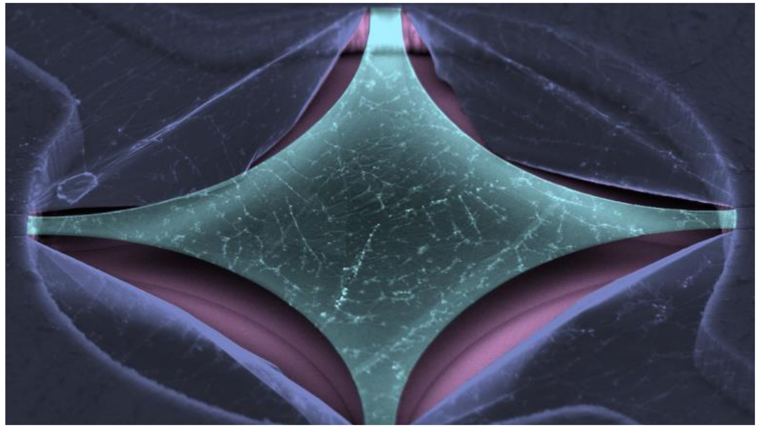
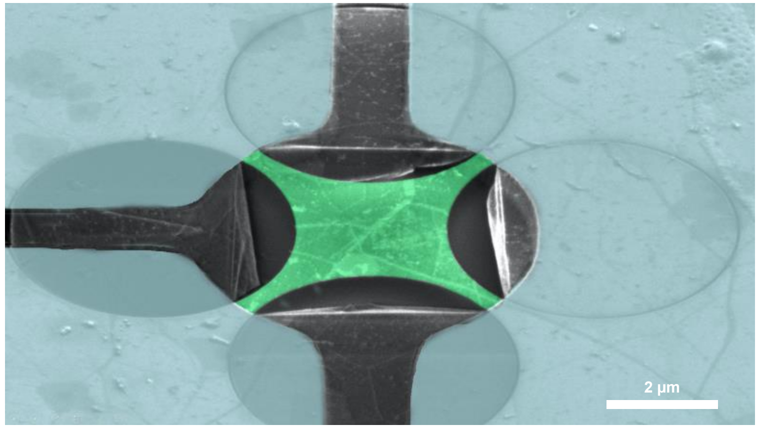
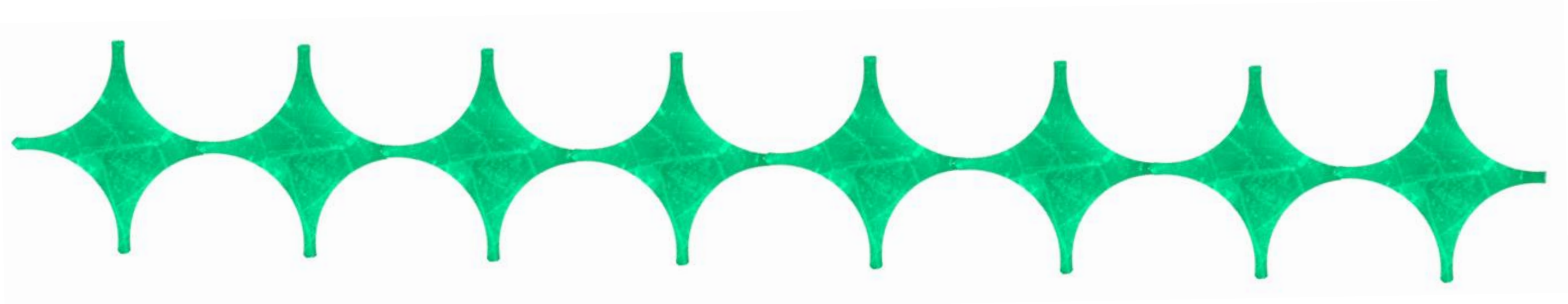


Potential experimental system

Graphene



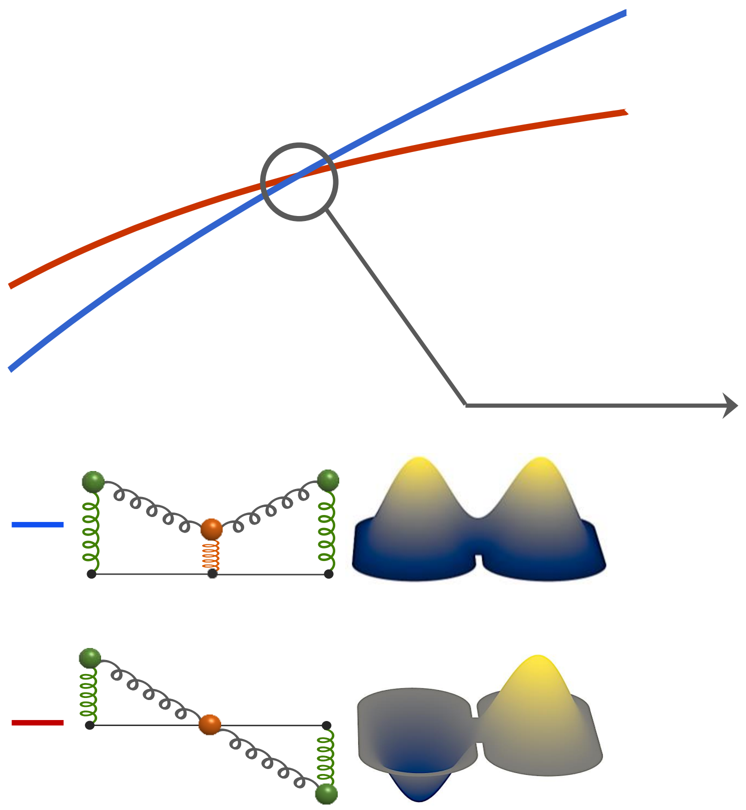




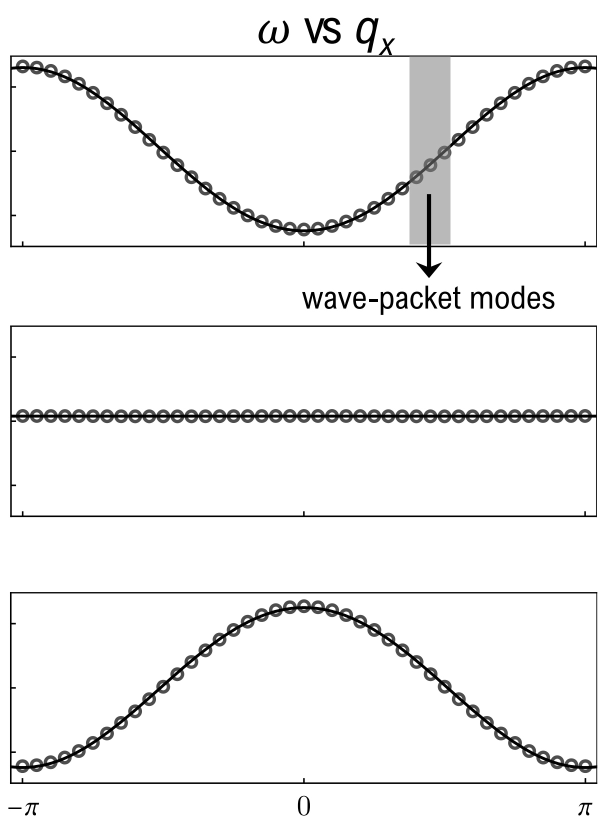
Benjamín Alemán group, University of Oregon

Summary in 1D

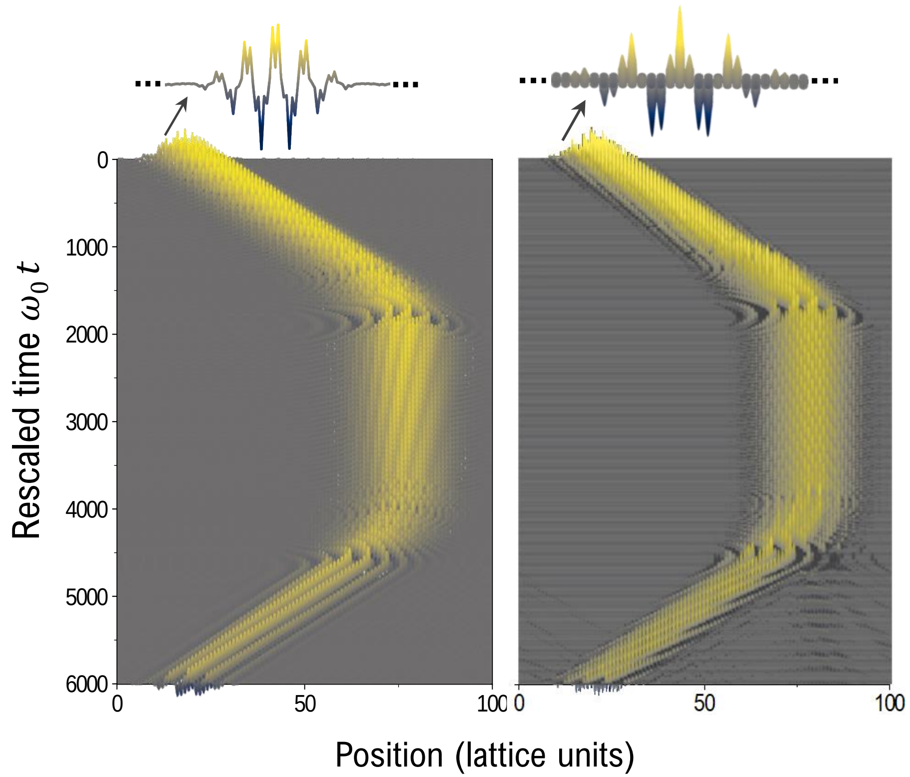
Vibrational modes crossing



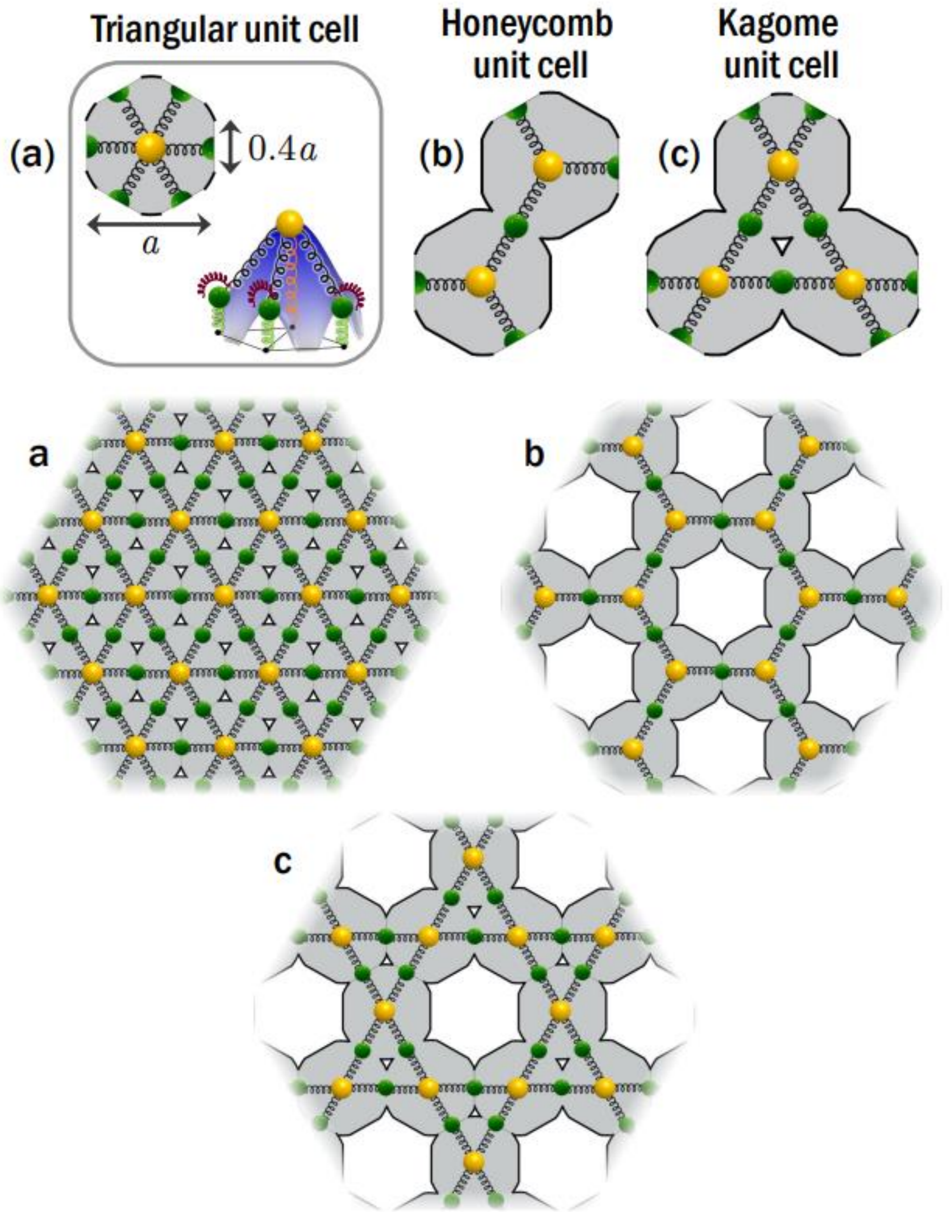
Changing dispersion character



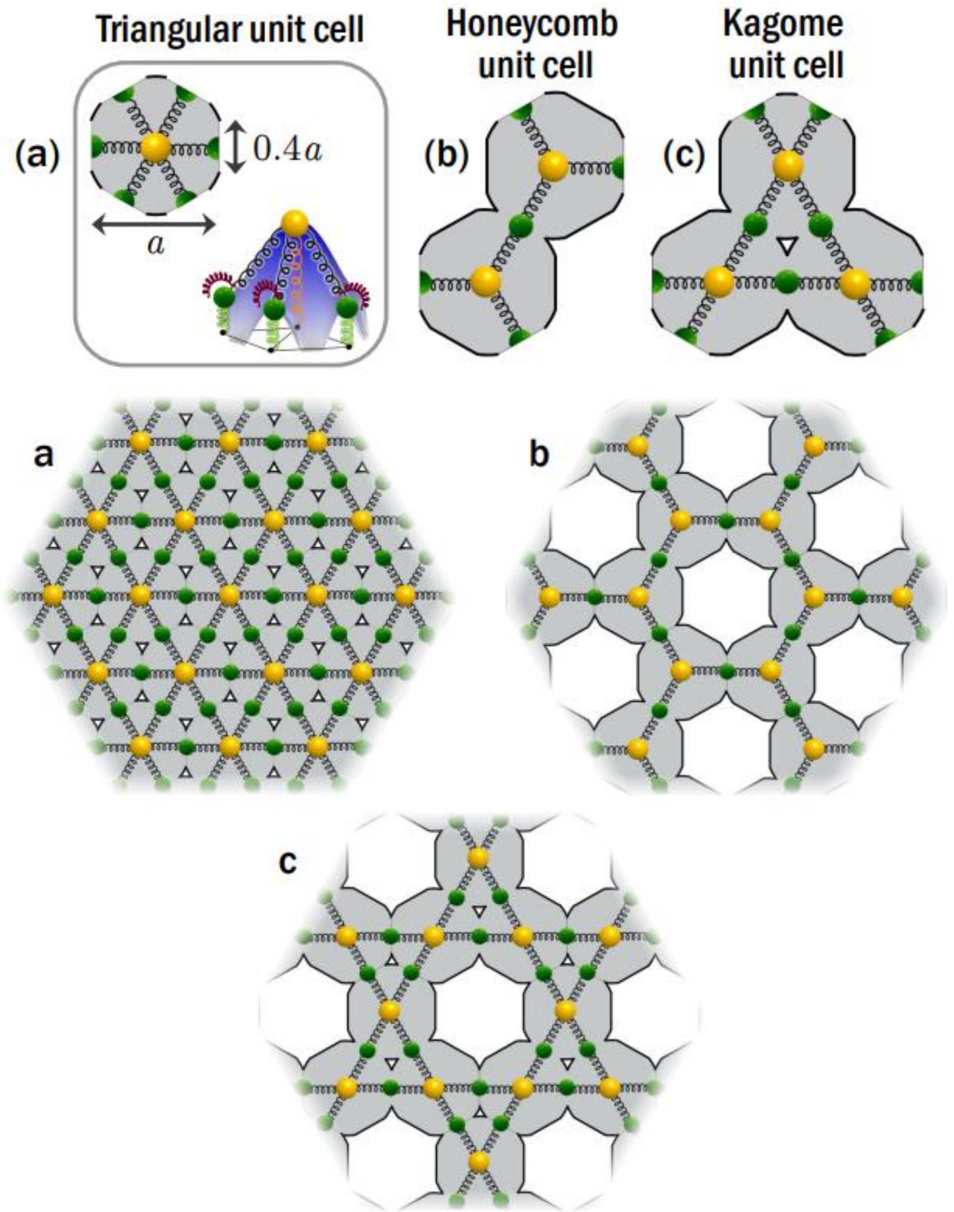
Stopping and reversing sound



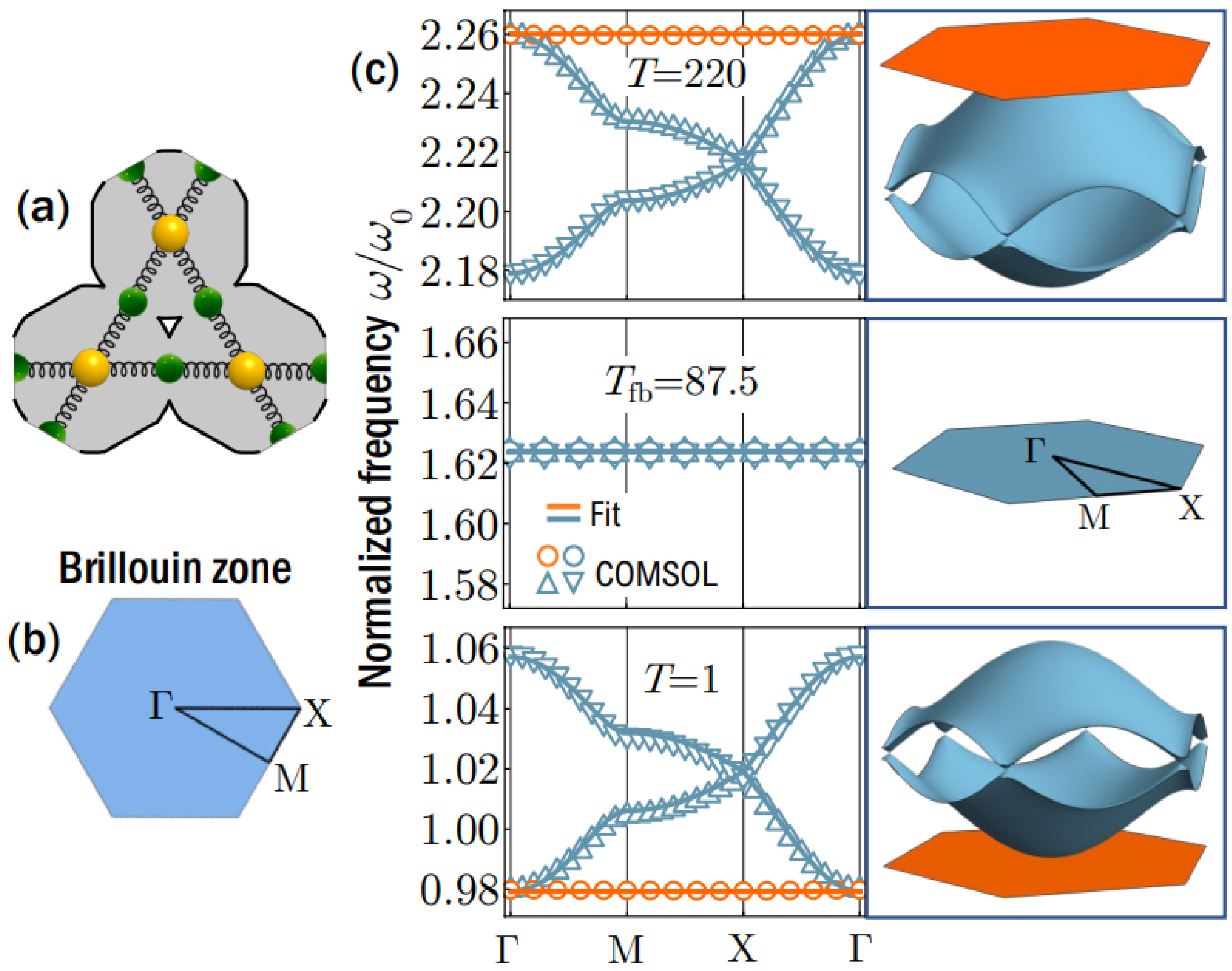
Singular flat band in 2D



Singular flat band in 2D



P. Karki and J. Paulose, *Physical Review Research,* 2023



Conclusions

- We created a custom model for coupled thin-plate elastic resonators
- We modified the bonding character of the ground state of a fourth order coupled thin-plate elastic resonator system via prestress modulation
- We imported solutions from eigenvalue problem to time-dependent studies to perform dynamical simulations