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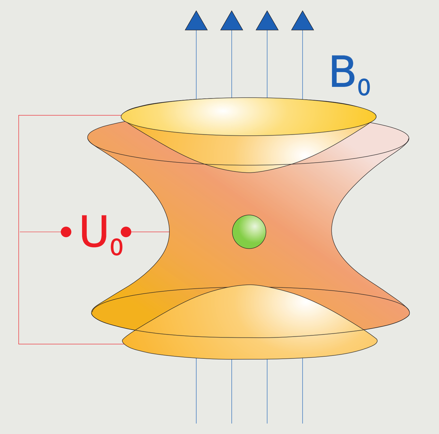
Image Charge Shift in Precision Penning Traps

M. Schuh[1], F. Heiße[1,2], J. Ketter[1], T. Eronen[3], F. Köhler-Langes[1], S. Rau[1], T. Segal[1], W. Quint[2], S. Sturm[1], K. Blaum[1]
[1] Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany
[2] GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstraße 1, 64291 Darmstadt, Germany
[3] University of Jyväskylä, P.O. Box 35 (YFL), FI-40014 University of Jyväskylä, Jyväskylä, Finnland

Penning traps

Usage and specifications

- Storage of single ion up to months [1]
- Vacuum better than 10^{-14} mbar
- Storage achieved by superposition of
  + Homogeneous magnetic field B_0
  + Electrostatic quadrupolar potential
- B_0 a few Telsa
- U_0 a few ten to 100 Volts
- Hyperbolic or cylindrical electrodes



Schematic Penning-trap electrodes

Ion motion

$$\begin{pmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{pmatrix} = \frac{qB_0}{m}\begin{pmatrix}\dot{y}\\ -\dot{x}\\ 0\end{pmatrix} + \frac{qU_0}{2md^2}\begin{pmatrix}x\\ y\\ -2z\end{pmatrix}$$

Parameters

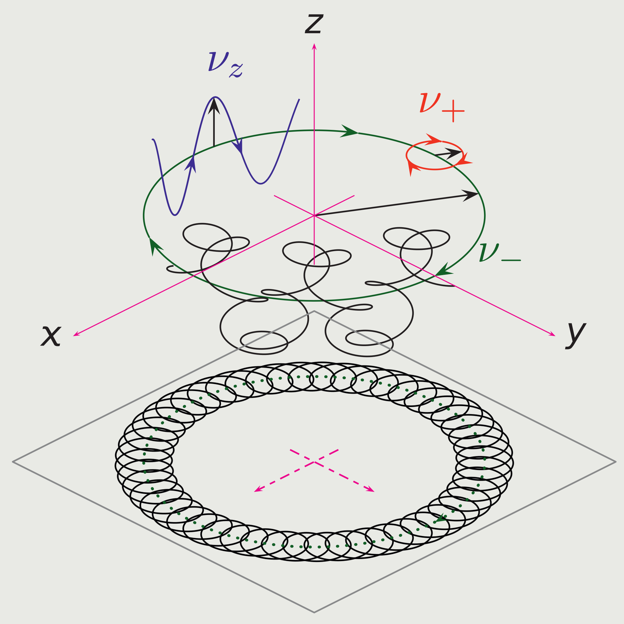
- q = charge of the ion
- m = mass of ion
- d = characteristic dimension of the trap

Equation solved by three independent motions

$$\nu_+ = \frac{1}{2}\left[\nu_c + \sqrt{\nu_c^2 - 2\nu_z^2}\right]$$

$$\nu_- = \frac{1}{2}\left[\nu_c - \sqrt{\nu_c^2 - 2\nu_z^2}\right]$$

$$\nu_z = \frac{1}{2\pi}\sqrt{\frac{qU_0}{md^2}}$$

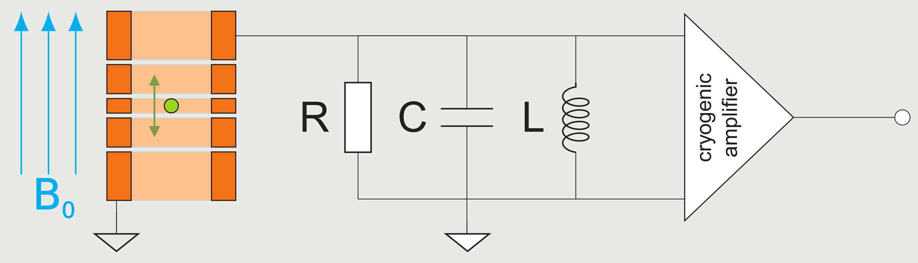


Eigenmotions in a Penning trap

The three eigen-frequencies can be combined to the free-space cyclotron frequency:

$$\nu_c = \frac{1}{2\pi}\frac{q}{m}B_0 = \sqrt{\nu_+^2 + \nu_-^2 + \nu_z^2} \quad [2]$$

Frequency detection



Frequency detection scheme

- Ion induces image charges in the electrodes [3]
- Current oscillation of fA
- Transformed by RCL resonator to voltage µV-level oscillation

Application and achievements

- Atomic mass ratio measurements; relative precision of some parts per trillion [4]
- *g*-factor measurement; relative precision of some parts per trillion [5]

Image charge shift

Theoretical description

- Induced charges create additional electric field $\boldsymbol{E}_{\text{image}}$
- Linear approximation is sufficient

$$\boldsymbol{E}_{\text{image}}(z,\rho) = n\left(\mathcal{E}_\rho \rho \boldsymbol{e}_\rho + \mathcal{E}_z z \boldsymbol{e}_z\right)$$

Parameters

- n = ion charge state
- $\mathcal{E}_\rho$ and $\mathcal{E}_z$ = gradient of linear field approximation

Effect on the eigen-frequencies

The eigen-frequencies introduced above are shifted as follows:

$$\Delta\nu_\pm = \mp n\frac{\mathcal{E}_\rho}{2\pi B_0}$$

$$\Delta\nu_z = -n\frac{q}{m}\frac{\mathcal{E}_z}{8\pi^2\nu_z}$$

$$\Delta\nu_c \approx n\frac{2\mathcal{E}_\rho + \mathcal{E}_z}{4\pi B_0}$$

Effects on the experiment

- Shift at a level of 10^{-10} - 10^{-12} (trap dependent)
- Largest systematic shift in all high-precision Penning traps. It dominates all other shifts by:
  + 30 times for the *g*-factor $^{28}Si^{13+}$ [5]
  + 120 times for the mass of the electron [6]
  + 3 times for the mass of the proton [4]
- Very hard to measure. Image charges are needed to be able to measure the frequency at all. Shift cannot be switched off or tuned.

Current knowledge

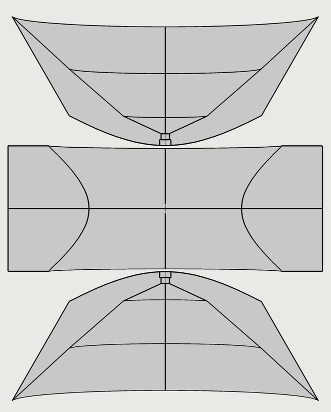
- Semi-analytical approach by J.V. Porto [7]. Hard to calculate and needs simplified trap geometry. Deviations to real case unknown.
- Theoretical approach by M. Kretzschmar and S. Sturm. Precision: 5% [5]
- Measurement by Zafonte et al. Precision: 4% [8]

Next step:
Finite element simulations!

Finite element simulation

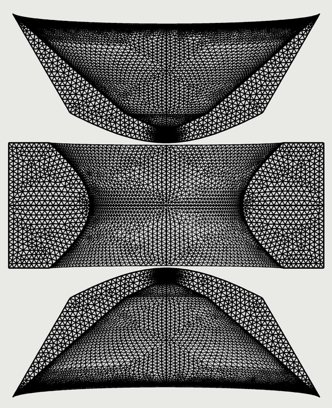
How to simulate?

1. Model the trap geometry.



Penning-trap geometry in software

2. Set all boundary conditions, place the ion and mesh the geometry.

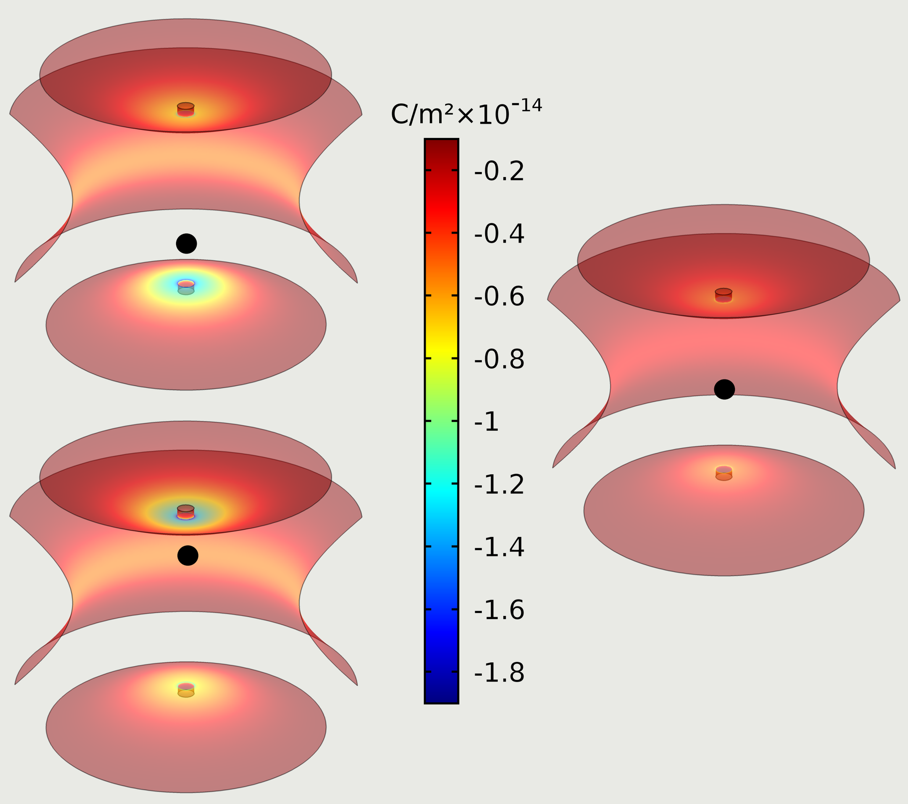


Meshed geometry

3. Simulate the current density distribution on the surfaces. Repeat this for several ion positions.

4. Calculate the field strength at the position of the ion.

5. Determine $\mathcal{E}_\rho$ and $\mathcal{E}_z$.



Surface charge density for different ion (in black) positions

How accurate is the simulation?

Test on analytical case
Test it on an "infinite" cylinder, where the analytic solution is known.

Result:
Simulation result deviates from analytical prediction at a relative 10^{-4} level.

Geometry uncertainties
Increasing the trap radius by 10 µm at an absolute radius of 5 mm.

Result:
Changes at a relative level of 0.7%!

Comparison to semi-analytical approach

- Confirmation of semi-analytical approach
- Necessary simplifications change result by up to 2.7%

Conclusion and outlook

Finite element simulation

- Agreement with all previous approaches
- Precision improved significantly to below 1%
- Already applied to many currently running Penning-trap experiment
- Limited by insufficient knowledge of geometry

Upcoming experimental data

- Novel measurement technique
- Using single ions having different masses
- Experimentally very demanding
  + Using same voltage for different masses
  + Needs two different axial resonators
- Precision below 4%
- In agreement with simulation

The understanding of the image charge shift has improved significantly. The simulation results are in excellent agreement with experimental data. An explicit measurement is very demanding and not all experiments can perform it. The finite element simulation can replace the measurement in most cases.

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