Use of COMSOL Multiphysics in High Voltage Electronic Transformers Design

A. Pokryvailo
Spellman High Voltage Electronics Corp.
475 Wireless Blvd, Hauppauge NY 11788, apokryva@spellmanhv.com

Abstract: Relation of the transformer electromagnetic field to the transformer equivalent circuit/s is revisited. Reference is also made to methods of measurement of the transformer parasitics. Difference in derivation of leakage inductance from static and frequency-domain analyses is illustrated. It is noted that the simplest stationary analysis is sufficiently accurate for practical purposes. A method of calculation of mutual inductance based on matched and opposite connection of windings is compared to those described in a COMSOL blog [1]. It is particularly useful for weakly coupled coils, e.g., in air-core transformers. Transformer optimization for operation in a resonant circuit is illustrated on an example of a Tesla-like transformer. Sweeping parameters, such as number of turns, windings' height and width, values of resonant capacitors, etc., in a wide range, we can arrive to a *desired* design point, which not necessarily is at resonance. For such simulations, Magnetic Field and Electric Circuit interfaces are mainly used. Distribution of electric field in multisectioned windings and parasitic capacitance of the transformer is calculated in Magnetic and Electric Field interface using RLC Coil Groups.

In practical use, the transformer is close to sensitive circuitry and/or metal construction elements. Then knowing fringe fields comes useful. Direct calculation of eddy current losses is compared to analytical estimates. A method of alleviating the induction heating effect is illustrated.

Keywords: COMSOL, high voltage, transformer, leakage inductance, inductive power transfer

1 Introduction

High Voltage (HV) Electronic Transformers operate at high frequency, typically in a range of tens-to-hundreds of kHz. Knowing and tuning their parasitic parameters is an important aspect at the design phase. Establishing thermal limits is important as well. Although a vast amount of analytical means is available for calculating parasitics[1], field simulations, especially in 3D, come to front as the ultimate tool for attaining high accuracy. Two simulation groups in transformer design are ubiquitous: electrostatic and magnetic, the latter usually using Magnetic Field formulation. The first is useful mostly for insulation design. (Transformer parasitic capacitance cannot be derived *directly* from the electrostatic field.) Unless non-linear media are involved, electrostatic simulations are rather trivial. The second group can yield magnetizing, leakage, and mutual inductances, and, of course, flux densities and losses in the ferromagnetic parts. Same, and parasitic capacitance, can be derived from electromagnetic (EM) simulation (Magnetic and Electric Field formulation). We focus on the second group.

In practical use, the transformer is close to sensitive circuitry and/or metal construction elements. Then knowing fringe fields comes useful. Direct calculation of eddy current losses is compared to analytical estimates. Methods of alleviating the induction heating effect are discussed.

2 Relation of the transformer EM field to the transformer equivalent circuit

It is common to represent a two-winding transformer[2] by two coils, *L1*, *L2*, coupled via mutual inductance *M* Figure 1a. Its well-known equivalent circuit with *lumped* parameters is shown in Figure 1b [4] (losses and parasitics capacitances are also depicted). Obviously, it presupposes a linear voltage distribution across the turns and is not applicable to the analysis of fast transients, when the time of propagation of the EM wave along the winding is less or commensurable with characteristic risetime.

Circuit Figure 1b covers a wide frequency range and is sufficient for most practical purposes of power electronics. However, an analytical treatment using this model is too complicated because of the high order of the corresponding differential equations. A simplified equivalent circuit Figure 1c works very well for the HV transformers with a *closed magnetic system*

[1] Literature on the subject is widely available; the author's "pre-Comsol" observations can be found in [3]; for more, see its references.

[2] It is recognized that the treatment can be extrapolated to multi-winding transformers, although this may be not trivial (see, e.g., [8].

(e.g., U and E-cores, popular with the HV transformers manufacturers), even for gapped cores, on condition that the magnetizing inductance is several times larger than the leakage inductance. The output capacitance *C* lumps together all parasitic capacitances including the rectifier parasitic capacitance).

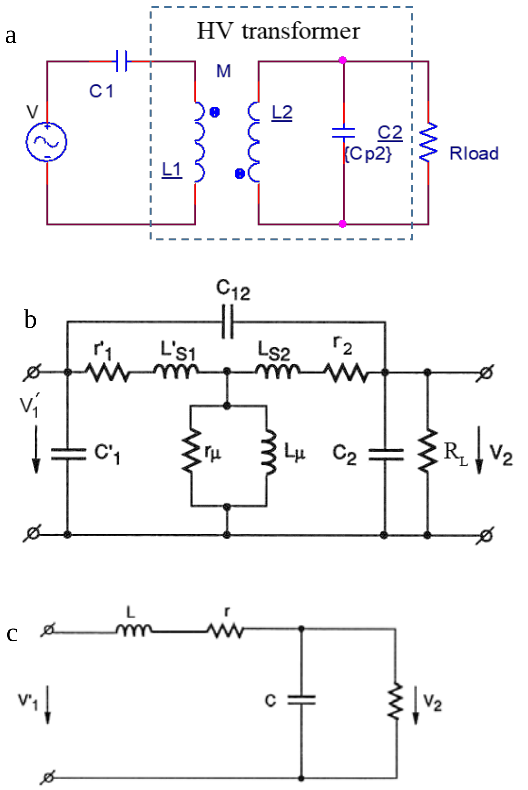


Figure 1. Circuit representation of high voltage transformer: a) coupled coils; b) full equivalent circuit; c) simplified circuit. All values are reflected to transformer secondary. Ls designate leakage inductances, C's – parasitic capacitances, resistors – losses in windings, insulation, and core.

The transformer equivalent circuits serve at least two purposes: a) for experimental derivation of the parameters and their interpretation; b) for circuit analysis of systems comprising transformers.

In our experience, circuit analysis of HV power converters, contingent on adequate calculation/measurement of the transformer parameters, is accurate, meaning that experimental waveforms match closely their simulated counterparts. Of the above parameters, leakage inductance and parasitic capacitance are the most important; the latter is also more difficult to both calculate and measure.

Since lumped parameters are only a reflection of a way the EM energy is stored in the transformer physical space, it is necessary to set properly the conditions for the energy generation in both field simulations and measurement setups.

2.1 Leakage inductance

It has been recognized very early (see, e.g., [5] that transformers have an "inductive voltage drop" associated with magnetic field "leaking" from the core. Thus, leakage inductance can be determined from *field distribution*, in principle, at *any* operation mode. One of the common methods of calculating leakage inductance is by the formula

$$L_s = \frac{2E}{I^2}, \tag{1}$$

where *E* is the energy stored *outside* the core at current I flowing in the corresponding winding. This method actually prescribes using equivalent circuit Figure 1c because the windings' currents are assumed equal when reflected to the same winding. Another implication of (1) is that only one Ls value is assigned to the transformer, lumping together Ls1 and Ls2. All this makes sense for transformers with good coupling, i.e., with closed magnetic systems. Using the term "leakage" inductance for loosely coupled coils, e.g., in air-core transformers, Tesla transformers being a good example, is counterproductive.

2.2 Parasitic capacitance

Likewise, parasitic capacitance Cp is calculated through electric field distribution, e.g., from the energy balance:

$$C_p = \frac{2E}{V^2}, \tag{2}$$

where *V* is the voltage across the winding. In HV transformers, virtually all electrostatic energy is stored in the space inside the secondary; a small part can be contributed to *E* by the field outside the secondary.

The field distribution needs to be a *dynamic* one, the implication being that Cp depends not only on the geometry and material properties, but also on the voltage distribution across the winding. Although this distribution may vary, especially at high harmonics, only one Cp value can be ascribed to the transformer when circuit Figure 1c is used. Usually, the voltage is shared between the turns, which greatly simplifies and enables efficient closed-form analysis. The accuracy of the latter is compromised, unfortunately, by difficulty in accounting for real wire geometry. Numerical analysis can easily cope with this problem. However, the irregularities of winding, impregnation, etc., may lead to large errors; Cp measurement remains the ultimate tool for determining this parameter.

2.3 Measuring selected transformer parameters

A ubiquitous method of measuring Ls is with an LCR meter by connecting it to one of the windings and shorting the other. (For HV transformers, we recommend shorting the primary.) This again implies using equivalent circuit Figure 1c. An implication for field analysis, if one desires a direct comparison with experiment, is setting the simulation similarly, namely, by driving the transformer at the same frequency, from the same winding, and shorting the second winding.

If Ls is known, Cp can be found by measuring series resonance frequency and using circuit Figure 1c.

Mutual inductance most reliably can be found from the relation

$$|M| = \frac{L_{eq1}-L_{eq2}}{4}, \quad (3)$$

where L_{eq1}, L_{eq2} are the inductances of the primary and secondary coils in series connection, in matched and opposite order, respectively. Note that a) L1, L2 knowledge is not necessary; b) for numerical field analysis, (3) requires setting the simulation different from [1].

3 Calculating leakage inductance in COMSOL

A model of a typical HV transformer with secondary well isolated from the low-voltage circuitry is shown in Figure 2. Here the windings are situated on the same leg of a U-core. Following classic methods of calculating Ls [5], [6], one can find it from stationary or frequency-domain analysis driving the windings by equal, but opposing currents. (A widely used formula for calculating Ls with this and additional simplifications is given below for reference:

$$L_{s2} = \frac{\mu_0 w_2^2 p}{h}\left(\Delta_{12} + \frac{d_1 + d_2}{3}\right). \quad (4)$$

Here w_2 is number of secondary turns, p is mean perimeter of *both* windings, h is mean height of windings, d_1, d_2 are the primary and secondary windings' thicknesses, respectively, and Δ_{12} is the distance between the windings).

Magnetic Field (MF) interface with multiturn coils is adequate for this analysis; core relative permeability can be constant and anything from hundreds to thousands. The bounding box can be quite small because the field is mostly constrained between the windings. (In analytical methods, *the entire* field concentrates between the windings.) Symmetry in xy and xz planes can be exploited to reduce the model size. Note that in this method, the magnetizing branch of the circuit Figure 1b is ignored, and again use is made of Figure 1c (neglecting Cp, of course).

Mimicking the experimental method of measuring Ls, the primary was driven by current with the secondary shorted by 0.0001Ω (MF coupled with Electric Circuit – EC- interfaces). At 20kHz, Ls2=4.63mH – not a significant difference compared to Ls2=4.71mH obtained in the stationary analysis[3]. We also did not find substantial difference between frequency-domain and stationary analyses. Since the latter is the fastest of all, there is no justification for a more complex MF-EC analysis for finding Ls of *closed-core* transformers.

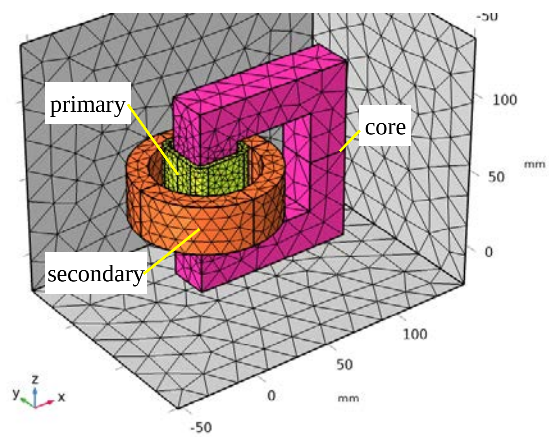


Figure 2. Core U100/57/25. w1=18, w2=270. From stationary analysis, Ls2=4.71mH. AC analysis at a number of frequencies, up to 50kHz, yields the same value.

For concentric windings whose thicknesses and separation are commensurable with the core, the difference between (4) and COMSOL simulations is substantial, 30-50%, in our experience[4]. Eq. (4), however, does not give a clue for the case of *not* overlapping windings, e.g., those sitting on the opposite legs of a U-core (see an example in Figure 3. In such transformers, leakage field is truly 3D, and simulations become even more useful. To allow the field to "leak", the bounding box must be large. Its size has major impact on calculated values. Table 1 gives Ls2 calculated for transformer Figure 3 and illustrates Ls drastic increase when windings are separated. The difference becomes larger with larger separation.

It is worth noting that if Ls is usually measured in "free space", and the bounding box is meant to imitate such a situation, in a real setting, the transformer may be very close to metal objects. They may reduce Ls considerably, and should comprise part of the model

[3] The measured value was Ls2=5.3mH, including ~0.3m primary leads. (There are no in-depth comparisons to experimental results in this paper.)

[4] Compare to a note from [5], p. 125: "The writer believes, however, that if *l* is taken equal to the mean length per turn of the windings … the Formula…will yield results sufficiently accurate for nearly all practical purposes".

(in frequency domain, at switching frequency) for better accuracy.

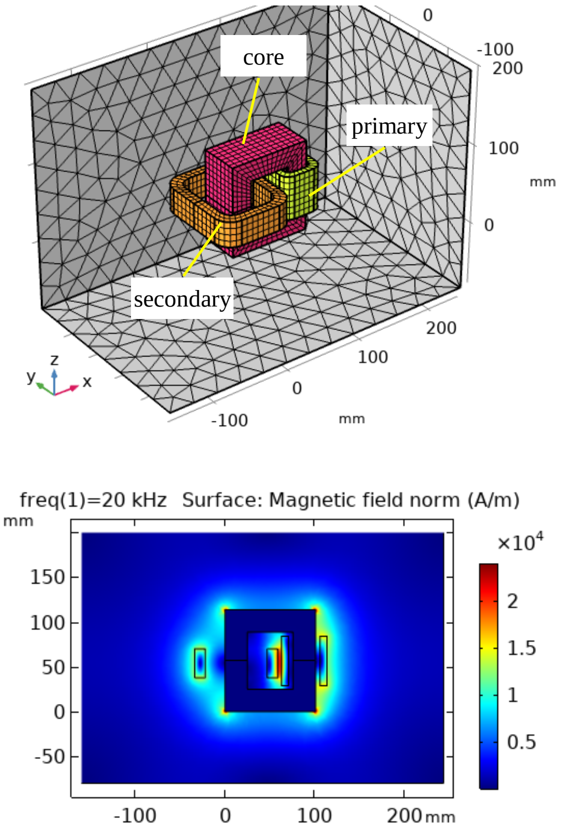


Figure 3. Windings on separate legs. Core 2xU100/57/25. w1=10, w2=320. Ls2=33mH. Primary driven by current, secondary shorted (MF-EC interfaces used).

Table 1. Typical dependence of Ls on the window size w (distance between the legs). Subscripts "diff" and "same" relate to windings situated on different legs, and same leg, respectively. From frequency-domain analysis.

w	Ls2, H	$Ls2_{dif}/Ls2_{same}$
0.051	0.0327	3.32
0.071	0.038	3.85
0.101	0.0454	4.60
0.151	0.0569	5.77

4 Transformer optimization for operation in a resonant circuit

Coupled field-circuit simulation makes optimization of transformers with weak coupling straightforward [2]. Figure 4 shows such a transformer and its COMSOL field model. For efficient operation, inductive leakage must be capacitively compensated as shown in Figure 1a. Circuit analysis may be quite efficient if the parameters of Figure 1a are known and well related to actual dimensions and materials' properties; this, generally, is not the case. In COMSOL, sweeping parameters, such as number of turns, windings' height and width, values of resonant capacitors, etc., in a wide range, we can arrive to a *desired* design point. In MF-EC interfaces, the simulation is fast and sufficiently accurate. Figure 5 gives an example of resonant curves showing classic frequency splitting (see details in [2]).

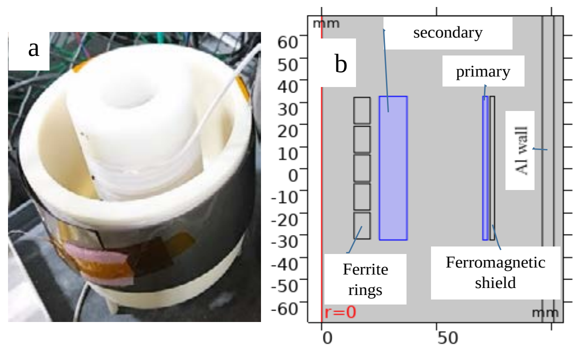


Figure 4. a - 1-kW, 20-kV transformer prototype. Outer cylinder carries primary winding with w1=50 turns Litz 420/38. It is covered by a ferromagnetic shield (amorphous metal tape). Inside cylinder (secondary bobbin) carries secondary winding (12 sections, number of turns varied from 2000 to 3500). Secondary can be lined with ferrite toroids (optional, accommodated inside secondary bobbin). b – cross-section showing homogenized secondary (COMSOL axisymmetric model).

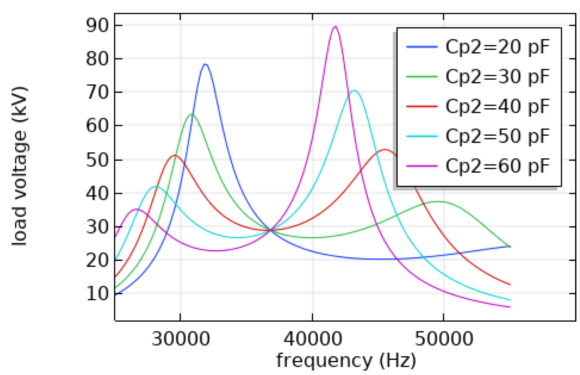


Figure 5. Frequency sweep for w2=2500, C1=47 nF, R_{load}=400 kΩ. Cp2 varied; with five ferrites. Single crossover point was observed also in circuit simulation and experiments.

Mutual inductance can be calculated as follows. Disabling EC, and driving the coils by equal currents, first in matched, and then opposite connection, we mimic (3). Calculation and measurement results are given in Table 2. The procedure of [1] yields close results.

Table 2. Mutual inductance M of transformer in Figure 4, calculated by (3) in COMSOL, and found experimentally, also by (3).

COMSOL			Experiment		
L_{eq1}	L_{eq2}	M	L_{eq1}	L_{eq2}	M
mH	mH	mH	mH	mH	mH
486.3	461.1	6.3	531.8	506.1	6.43

The MF formulation disregards in-plane displacement currents, and thus does not enable calculation of the coil parasitic capacitance. MEF-EC allow modeling voltage distribution along the winding, and calculating its *dynamic* capacitance. This requires detailed modeling of the secondary winding geometry. Sweeping parameters as described above becomes time-consuming. However, simulation around "optimal" point determined in a simpler MF formulation is feasible.

Figure 6 illustrates voltage distribution in a 12-section secondary in an air-core transformer for the same parameters as in Figure 4. Both windings are modeled as RLC coil groups. The voltage is induced uniformly along the secondary winding. At several hundreds of kHz the winding starts self-resonating. The values of inductance at low frequency match their experimental counterparts; calculated by (2), capacitance is ~5pF, slightly higher than the experimental value of 3.5pF.

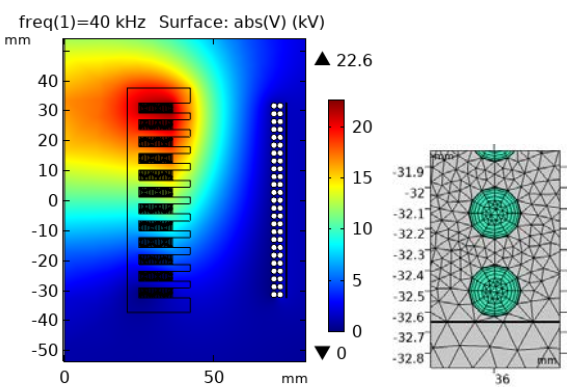


Figure 6. Model of air-core 1-kW, 20-kV high-potential transformer in axisymmetric approximation. Primary (outside) has 50 turns. Secondary, inside, has 12 sections, 2496 turns. Detailed structure allows calculating self-resonances, parasitic capacitance, etc. In this simulation, transformer operates in a resonant circuit; Rload=400kΩ.

5 Transformer fringe fields

In practical use, the transformer may be close to sensitive circuitry and/or metal construction elements. Then fringe fields, both magnetic and electric, may have major impact on the system performance. We focus here on magnetic fields making use of the following three (synergistic) examples: a) attenuation of fringe field by metal enclosure (see also Figure 4b); b) influence of metal parts on Ls; c) induction heating effects.

Figure 7 shows an HV transformer on a U-core inside an aluminum (Al) envelope. The windings on separate legs create large leakage fields spreading far and wide. The envelope would effectively contain high frequency EM field. Eddy currents in it generate heat and tend to decrease leakage inductance. Usually, eddy current simulations do not converge well, and simulations take long time, so we exploited symmetry and modeled only ¼ of the space. It was assumed that the windings have equal Ampere-turns. At 40kHz, I1=10A_{peak}, the envelope loss is ~7W, whereas Ls2 decreased from 0.3H to 0.26H. The magnetic field is virtually screened by the envelope at this frequency Figure 8.

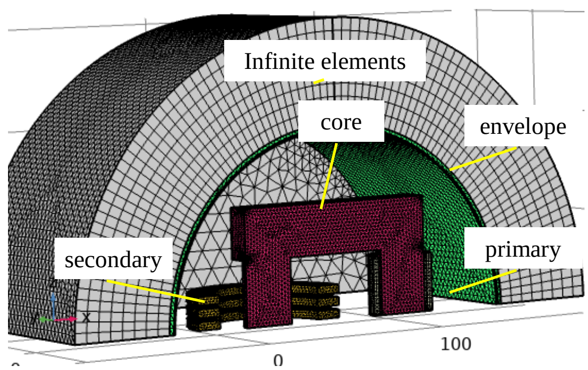


Figure 7. HV transformer on a U-core inside Al envelope. w1=35, w2=1050. In view of symmetry, ¼ of geometry is modeled.

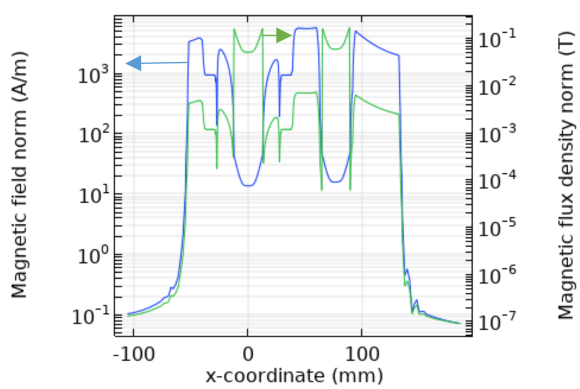


Figure 8. Flux density and field along axis of symmetry (x-axis) for system **Figure 7**.

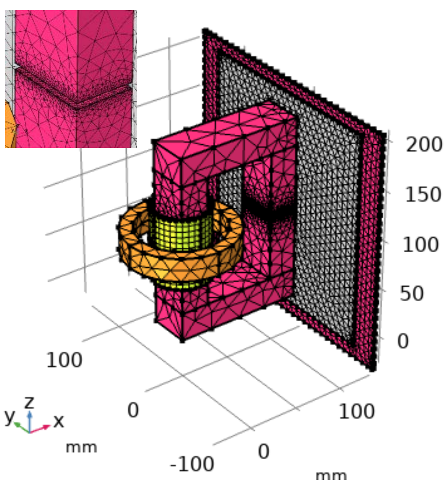
Another example presents a transformer with a gapped core near a steel plate Figure 9. At 50kHz, a ~15kW-sized transformer generates >300W losses in steel. Field plots and analytical estimates help to understand this phenomenon. An EM wave with field strength H incident on a conducting plate with area A generates loss

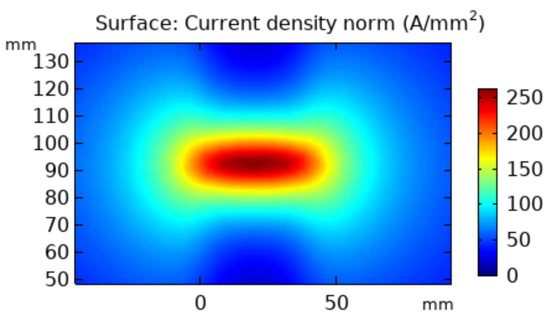
$$P := A \cdot \sqrt{\frac{2 \cdot \pi \cdot f \cdot \mu_0 \cdot \mu_r}{2\gamma}} \cdot \frac{H^2}{2} \qquad (5)$$

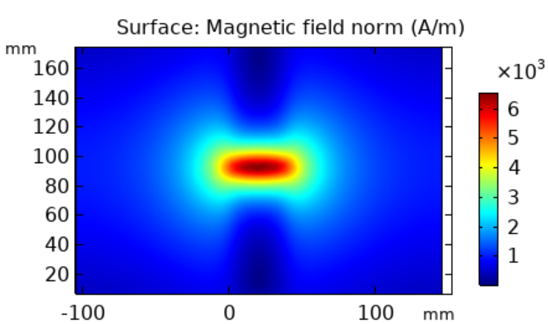
where μ_r, γ are relative permeability and conductivity of the plate. Adopting the same values of the parameters as in the simulation (A and H are estimated from the H-plot Figure 9),

$\gamma := 4 \cdot 10^6$ $\mu_r := 1000$ $A := 15 \cdot 15 \cdot 10^{-4}$ $f := 50000$ $H := 2000$

we calculate P=316W against 340W obtained by volume integration.







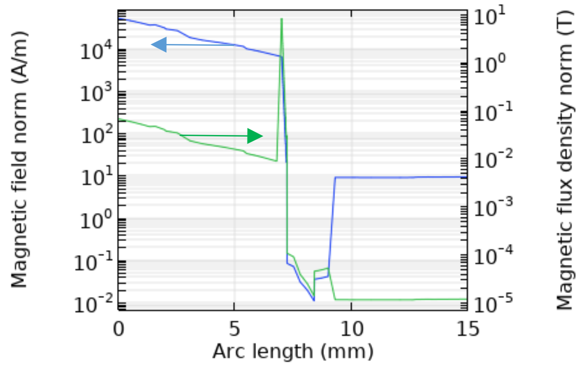
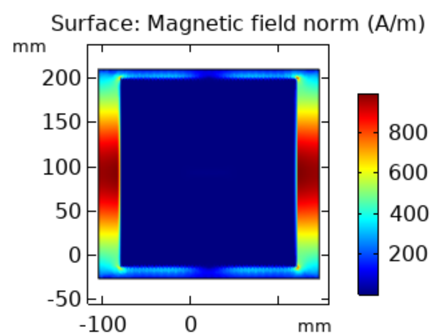
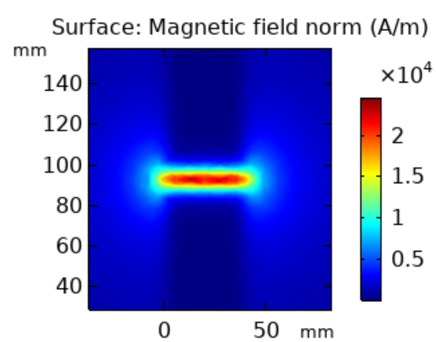


Figure 9. Transformer with core gapped 3.125mm each leg near a steel wall; latter may be lined with 2mm-thick Al. Distance from outer leg to steel is 7mm. In these simulations, Al is disabled. Loss in steel is 340W. H, B are plotted along Cut Line starting flush with core from middle of the gap, outward to steel.

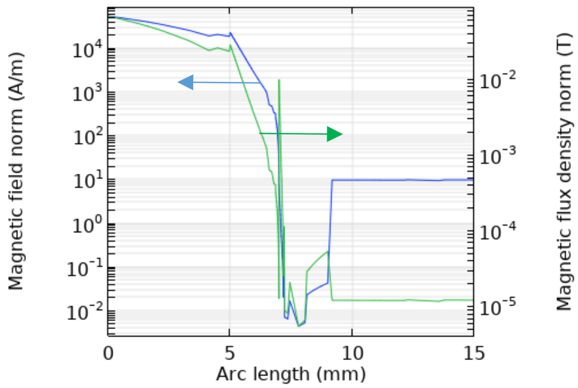
If steel is lined with a 2-mm-Al plate, overall losses decrease dramatically, in steel to 24W, albeit adding 11W in Al. The shielding effect is well known (see, e.g., [7]); it also can be predicted from (5). Figure 10 illustrates field attenuation in 2mm of Al.



a



b



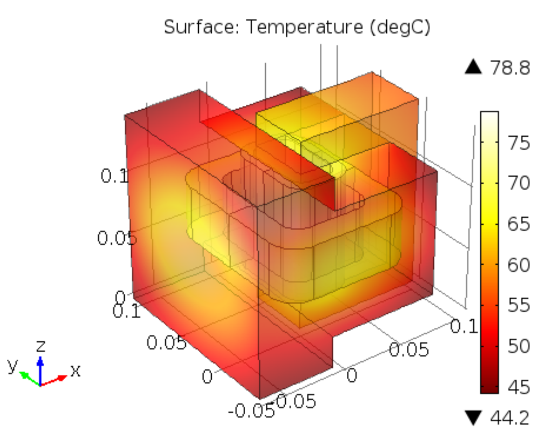
c

Figure 10. Steel wall lined by 2-mm-thick Al plate. Loss 24W in steel, 11W in Al. a, b – fields incident on steel and Al plates, respectively; c – same as in Figure 9 (note field attenuation in 2mm of Al).

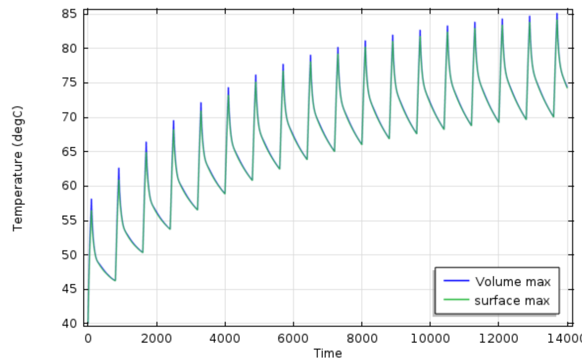
6 Transformer thermals in intermittent operation

This section deviates from the main course of this report – EM simulations. Ultimately, as in many other technical systems, power delivery of HV transformers may be limited by overheat. If stationary heating is relatively easy to simulate or assess on the base of previous experience, transient heating is more challenging, especially in intermittent mode of operation. We found COMSOL particularly useful for this purpose. In circuit and/or EM simulations, we can determine losses, and using the same geometry, with minor modifications, run thermal analyses.

Figure 11a shows a transformer similar to that of Figure 3 (windings are situated on the same leg) encapsulated in solid dielectric. To the core and the windings are assigned losses calculated for 50-kW operation. The duty cycle is 100s on, 700s off. Temperature rise in time is illustrated by Figure 11b.



a



b

Figure 11. 50kW high line 100s on 700s off. T0=20degC and 40degC.

7 References

[1] Analyzing Mutual Inductance in Different Coil Arrangements, August 19, 2015 https://www.comsol.com/blogs/?s=mutual+inductance

[2] A. Pokryvailo, “Coupled Magnetic Field-Circuit Analysis of Inductive Power Transfer in High-Potential Transformers”, presented at Wireless Power Week, 17-21 June 2019, London.

[3] A. Pokryvailo, “Accounting for Parasitic Parameters in High-Voltage Converters”, Proc. Annual Conf. of the Electrical and Electronics Engineers Society, Eilat, 14-16 November, 2007, available at https://www.researchgate.net/publication/262068821_Accounting_for_Parasitic_Parameters_in_High-Voltage_Converters

[4] L.J. Giakoletto, *Electronic Designer's handbook*, McGraw Hill, N.Y., 1977, 2240pp.

[5] A. Still, *Principles of Transformer Design*, NY, Wiley, 1919, 216pp.

[6] A. I. Voldek, *Electrical Machines*, 3rd ed., Leningrad, Energia, 1978, 832pp. (In Russian).

[7] J. Turowski and M. Turowski, *Engineering Electrodynamics*, CRC Press: Boca Raton, USA, 536pp, 2014.

[8] L. B. Leites, A. M. Pinzov, *Equivalent Circuits of Multi-winding Transformers*, Energia, Moscow, 1974, 192pp. (in Russian).

8 Acknowledgements

The author thanks Dr. W. Frei of COMSOL for his help in setting the model Figure 6, and Spellman High Voltage Electronics Corp. for supporting this work.