Towards Estimating Diffractive Sidewall Scattering Loss in Light Pipes

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Introduction

The impact of surface roughness on the optical transmission of light pipes has been addressed using a geometrical optics approximation (GOA) [1]; however, a basic estimate for diffractive losses remains to be included. Using COMSOL®, we compare 2D Wave Optic Simulations for diffractive efficiencies to those predicted by non-paraxial scalar theory [2] for sinusoidal phase gratings.

Glass light pipes have been fabricated using femtosecond laser irradiation followed by chemical etching [3]. After etching, the surfaces are rough and may require subsequent polishing to achieve optical quality smoothness. The degree of smoothness will impact the transmission, so simulation work aims to understand the tradeoffs in surface roughness parameters and induced loss. The distribution of ray angles changes upon intersection with the scattering sidewall surface. The sidewall scattering loss results from incidence angles that are smaller than the critical angle.

Simulations using the GOA yield the same transmission versus ray incidence angle for random surfaces with different RMS heights (σ's) but the same root-mean-square (RMS) slope (m_{rms}=0.01) as shown in Fig. 1. Different surface RMS slopes (m_{rms}=0.02) change the transmission curve. The GOA does not predict any significant wavelength dependence. Thus, a model including diffractive effects would be helpful for analyzing experimental results.

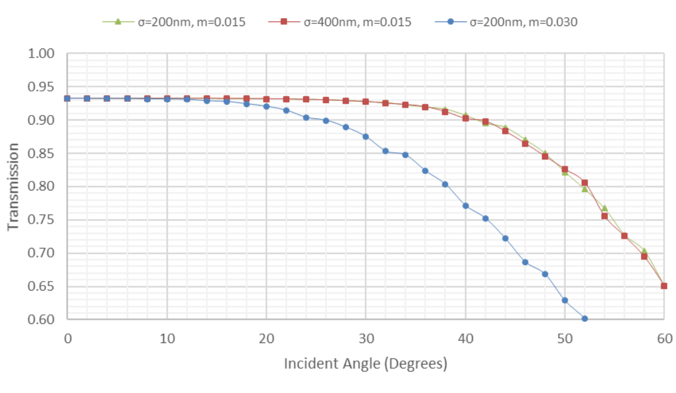


Figure 1. A GOA simulation for sidewall scattering loss for a 1mm^2 by 50mm-long fused silica light pipe in air [4]. RMS slope=m.

While rough surfaces can be simulated and the diffraction precisely modeled, full vectorial electromagnetic solutions are time consuming and solutions are needed for a large ensemble of surfaces. Thus, our goal is to modify the GOA simulation to include diffractive effects on each ray intersection with the (core-to-cladding) interface. A simple transfer function approach for incident to reflected ray angles is desired. By starting with a sinusoidal phase grating, we can extend the result by Fourier series expansions to address random surface variations.

A Sinusoidal Phase Grating - Analytic Solution

Harvey and Pfisterer [2] describe a nonparaxial scalar diffraction theory with an analytic formula for the case of a sinusoidal phase grating. The nonparaxial scalar solution has been used more generally for random surfaces where close comparisons have been established to full vectoral electromagnetic TE simulations. The grating equation for the mth-order is given as follows [2]:

$$\beta_m + \beta_i = \frac{m}{\hat{d}}$$

where $\beta_i = sin\theta_i \ and \ \beta_m = \sin\ \theta_m$. The normalized peak-to-valley grating height and period are given by $\hat{h} = \frac{h}{\lambda}$ and $\hat{d} = \frac{d}{\lambda}$, respectively. The efficiency for each order is calculated using mth-order Bessel functions of the first kind and normalizing to the sum over all propagating orders as follows:

$$\eta_m = \frac{J_m^2\left(\frac{a}{2}\right)}{\sum_{m=min}^{max} J_m^2\left(\frac{a}{2}\right)}$$

where the Bessel function argument is given by

$$\frac{a}{2} = \pi\hat{h}\,[\cos(\theta_i) + \cos\,(\theta_m)].$$

For a given set of grating parameters and wavelength, the efficiency η_m into each order is quickly calculated in MATLAB®.

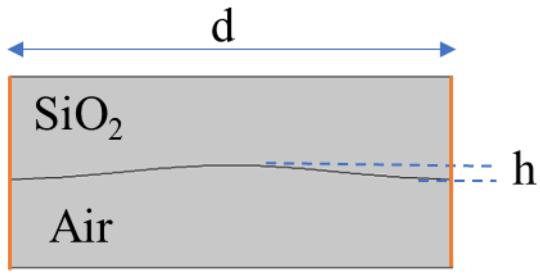
As previously noted, an important metric for estimating sidewall scattering loss using the GOA is the RMS surface slope [1]. The RMS slope can be calculated for random and deterministic surfaces. For a sinusoid with period d and peak-to-valley height h, the RMS slope is denoted by [5]

$$m_s = \frac{\pi}{\sqrt{2}}\frac{\hat{h}}{\hat{d}}$$

The maximum slope is $\sqrt{2}$ times the RMS slope. For calibration, relatively smooth surfaces have RMS slopes of 0.01 or less. Light pipe scattering losses increase as the RMS slope increases under the GOA. The following simulations aim to evaluate whether this metric is sufficient to capture diffractive effects or whether other parameters are required.

TE Simulation Results

A single core-to-cladding interface is simulated, with n_1=1.46 and n_2=1.0, using the Wave Optics Module (2D simulation, TE-polarization) to calculate the diffraction order efficiencies for a range of parameters. The critical angle is $\theta_c = 43.23°$. Two examples are shown in Fig. 2 for a free space wavelength of 550nm, m_s=0.044, and $\hat{h}$=0.106 and 0.212. Clearly, the reflected ray distribution is quite different for each $\hat{h}$ value, with more specular reflection R(0) for the lower value. Periodic boundaries and ports are used with the wave incident from the silica to air interface.



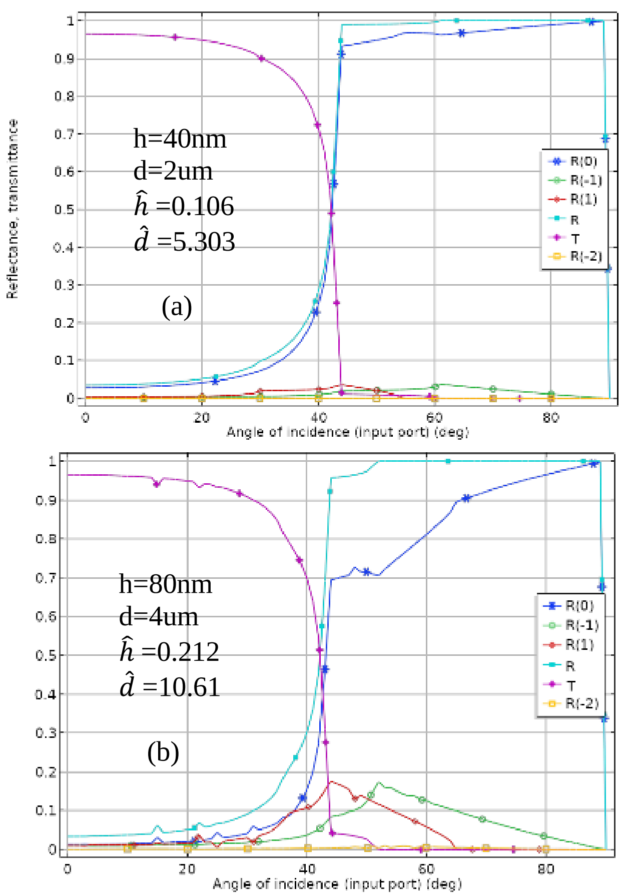


Figure 2. COMSOL® Wave optics TE simulation for sinusoidal 2D surface at a glass-air interface, showing the 0th, +/-1 and -2nd diffraction orders for reflection. The total reflectance R and transmission T are also plotted.

Non-paraxial scalar calculations are shown in Fig. 3a for $\hat{h}$ = 0.318 and Fig. 3b for $\hat{h}$ =0.212. Both plots use the same slope m_s=0.044 and free space wavelength of 550nm. Fresnel reflection losses are not included in Fig. 3, just the efficiency into each diffraction order. The sign convention for plus and minus orders is different between the COMSOL® diffraction order ports and the equations from [2]. Using the sign convention from [2], the cutoff for the m=-1 order is given by

$$sin\ \theta_i|_{-1cutoff} = 1 - 1/\hat{d}$$

For angles larger than this cutoff, only the +1 and 0th orders are diffracted. The cutoffs are 69.6° and 65° for Fig. 3a and b, respectively.

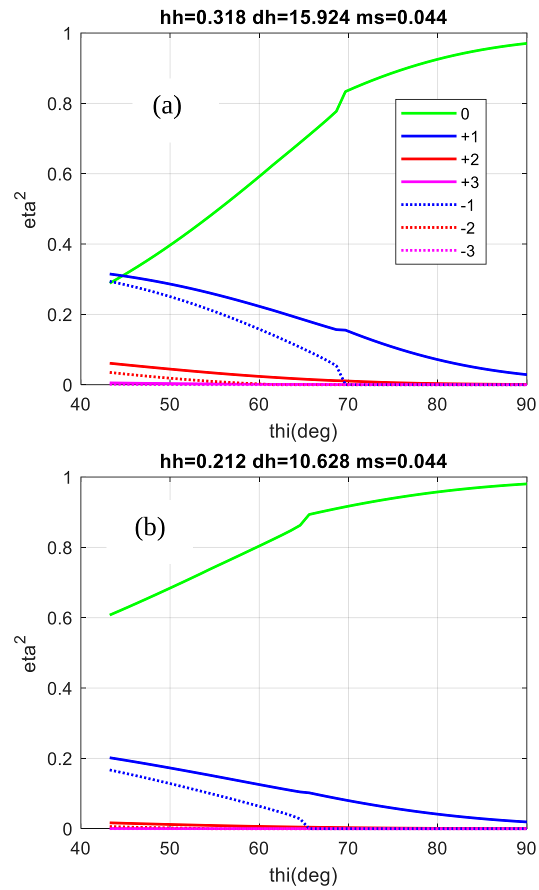


Figure 3. Nonparaxial scalar diffraction efficiency calculations (hh= $\hat{h}$, dh= $\hat{d}$), up to 3rd order, neglecting Fresnel reflection losses.

In Fig. 4, the efficiency for each order was multiplied by a TE Fresnel reflection term given by $\sqrt{R(\theta_0)R(\theta_m)}$. While good agreement was obtained for the other orders, the 0th-order predicted by the Wave Optics simulations were higher than the scalar theory calculation. An empirical normalization factor was found for the 0th-order, as follows:

$$\eta_0 \cong \frac{J_0^2\left(\frac{a}{2}\right)}{\sum_{m=min}^{max} R(\theta_m)^{0.75} J_m^2\left(\frac{a}{2}\right)}$$

This empirical "normalization" for the 0th-order, and standard normalization [2] for the other orders, provided good agreement between the Wave Optics and scalar theory results for each diffraction order as well as the total reflectance, as seen in Fig. 4. The empirically- determined normalization factor is the geometric mean of $R(\theta_m)$ and $\sqrt{R(\theta_m)}$. Note that $R(\theta_m) = 1$ for m=0 and -1, and is less than unity for m=+1 in the simulation results shown.

An example of calculations for the total reflectance due to diffractive losses using the nonparaxial scalar approach is shown in Fig. 5 for h=80nm and d=4μm at three different wavelengths. While the loss is much higher for incidence angles near θ_c for the shortest free-space wavelength λ=367 nm, it is higher at 550 nm and 825 nm at various larger incident angles, θ_i. Thus, having a quick calculation tool is helpful in estimating the total reflectance versus incidence angle for various combinations of slope and $\hat{h}$.

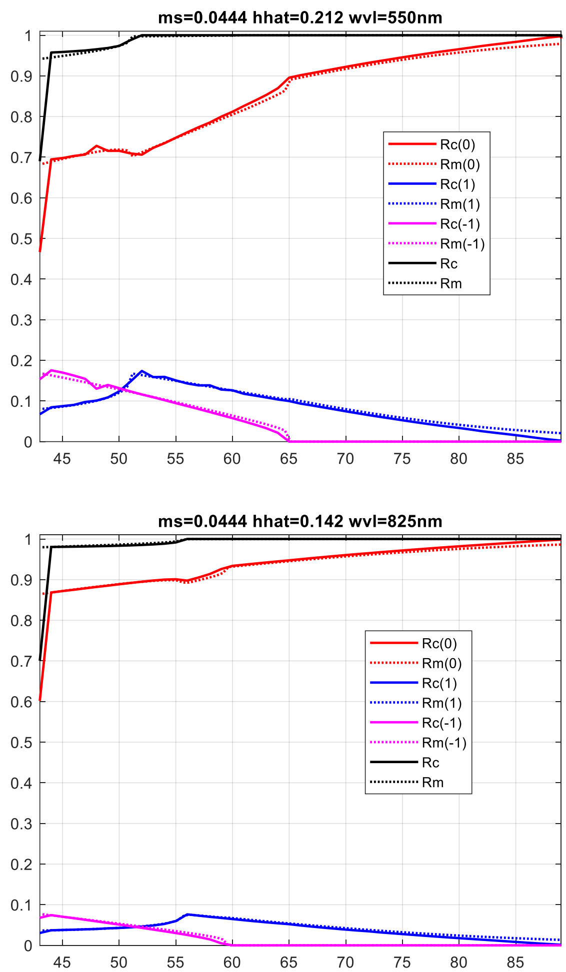


Figure 4. Comparison of Wave Optics and nonparaxial scalar TE diffraction estimations. Subscripts: c=COMSOL® Wave Optics, m=MATLAB® (nonparaxial scalar theory).

TM Simulation Results

An example TM response compared to the TE response from a 2D Wave Optics simulation is shown in Fig. 6 for the grating properties of d=4μm and h=80nm at a free space wavelength of 550nm. The main difference is a decrease in the +1 order and increase in the 0th order until the +1 order meets the total internal reflection (TIR) condition.

When the TM Fresnel reflectivity and the same empirical normalization procedure as previously noted is used with the scalar theory, a reasonably good agreement with the Wave Optics simulations is obtained, as shown in Fig. 7 for free space wavelength 550nm and 825nm. The transition regions, where one order becomes TIR or cuts off, do not have as good of an overlap as in the TE case.

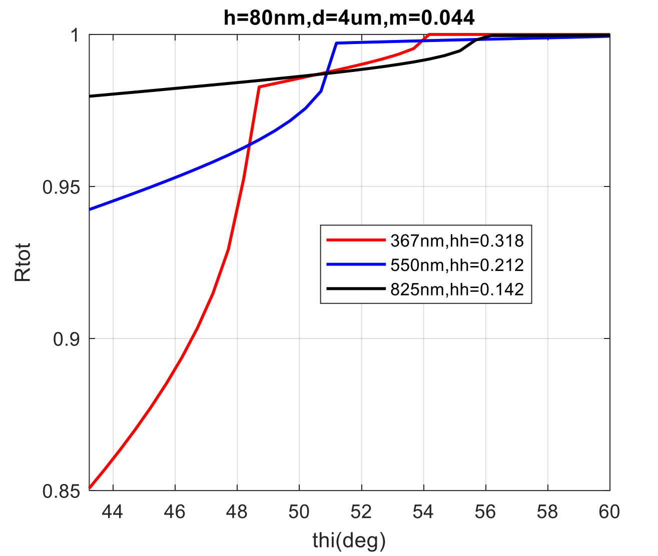


Figure 5. Reflection losses (i.e. when $|\theta_m| < \theta_c$) for 367nm (red), 550nm (blue) and 825nm (black) assuming d=4μm and h=80nm.

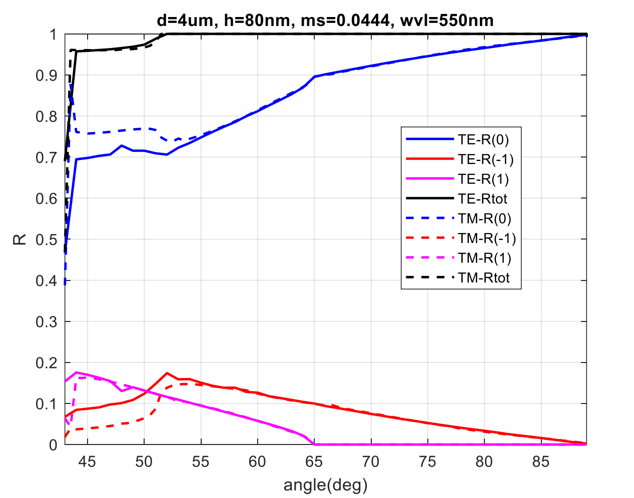


Figure 6. Comparison of TE and TM diffracted efficiencies using the COMSOL® Wave Optics module.

Conclusions

Insight into the diffractive scattering losses, and thus wavelength dependence, at a dielectric interface is gained by simulating a sinusoidal phase grating. The full vectorial electromagnetic simulations compared well to the nonparaxial scalar diffraction calculations using an empirically determined normalization of the 0th-order.

Unlike the GOA, both the RMS slope and the normalized height $\hat{h}$ are required to calculate the efficiencies into each diffraction order. Based on the nonparaxial scalar theory, any combination of wavelength and d that give the same slope and $\hat{h}$ will have the same diffraction efficiencies for its orders.

The nonparaxial scalar diffraction theory provides a simple approach for estimating the scattering angle distribution for light pipe sidewall scattering loss calculations. In future analyses, a Fourier series of grating amplitudes and phases will be used to model a desired (e.g. rough) surface and the sidewall scattering losses predicted for a large number of ray intersections with the core-to-cladding interface to model a realistic light pipe.

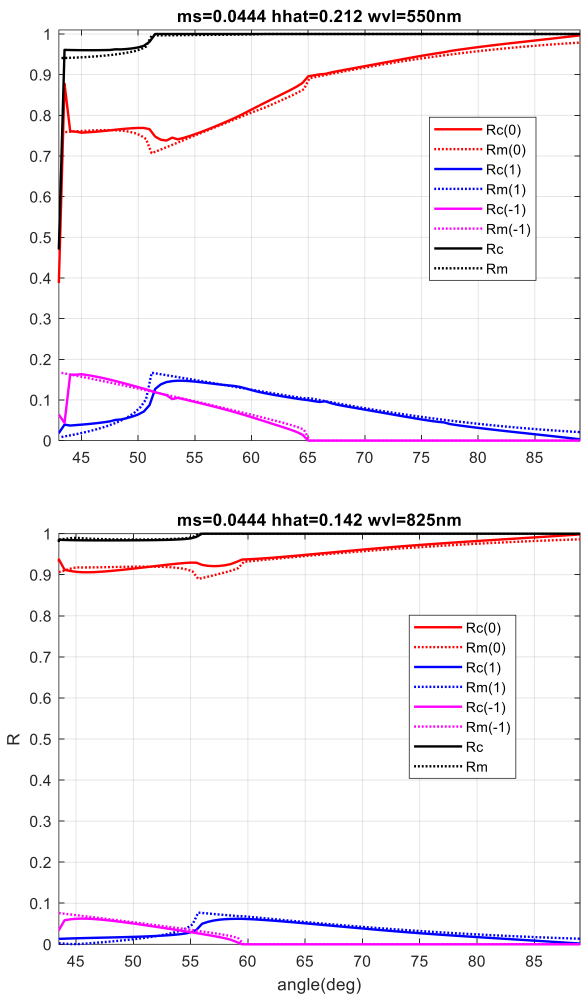


Figure 7. Comparison of TM diffracted efficiencies from the Wave Optics module and the scalar theory. Subscripts: c=COMSOL®, m=MATLAB®.

References

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