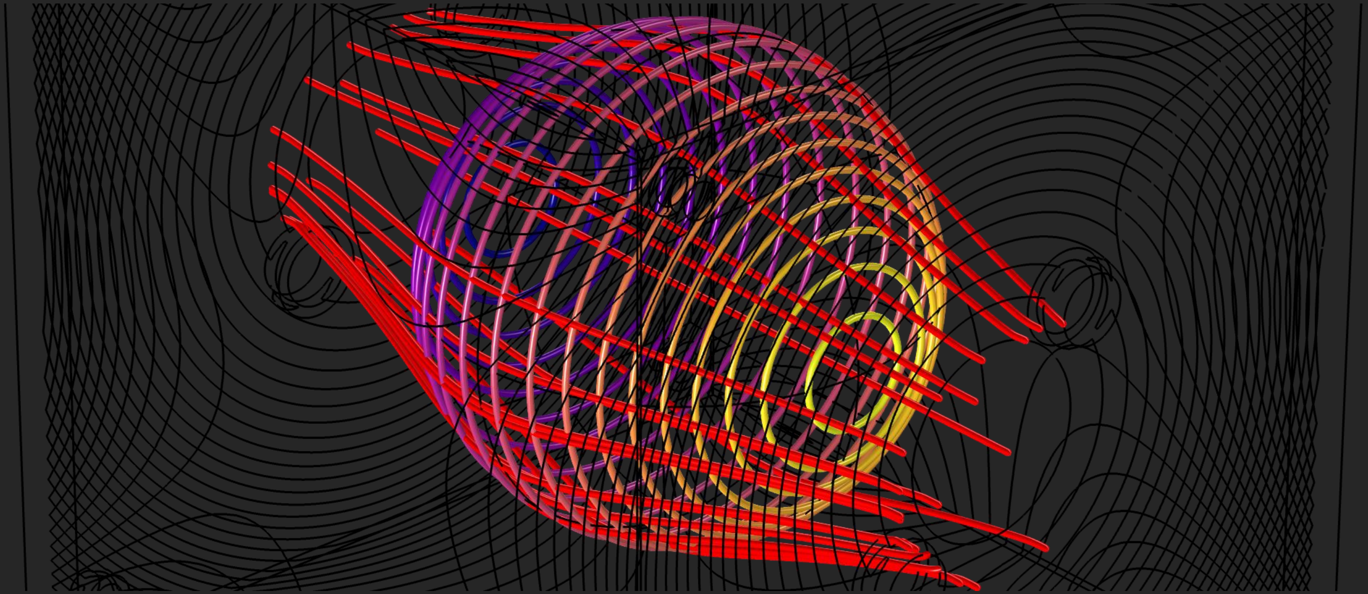


Coil Solver: Gradient Optimization with Scalar Potential Techniques

Austin Reid 2020.10.07 COMSOL Conference: Electromagnetics

INDIANA UNIVERSITY BLOOMINGTON

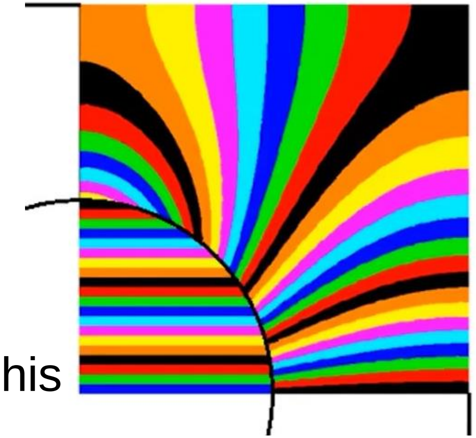
Scalar Potential Coil Design

In free space:

$\vec{\nabla} \times \vec{H} = 0 \qquad \vec{\nabla} \cdot \vec{H} = 0 \Rightarrow \vec{H} = -\vec{\nabla} u \qquad \nabla^2 u = 0$

Surface discontinuity:

$\hat{n} \times -\vec{\nabla}(u_{out} - u_{in}) = \vec{K} \qquad -\Delta u = I$



Thanks to Chris Crawford. For more detail, see one of his talks here: https://youtu.be/LTuk-sz-ApE

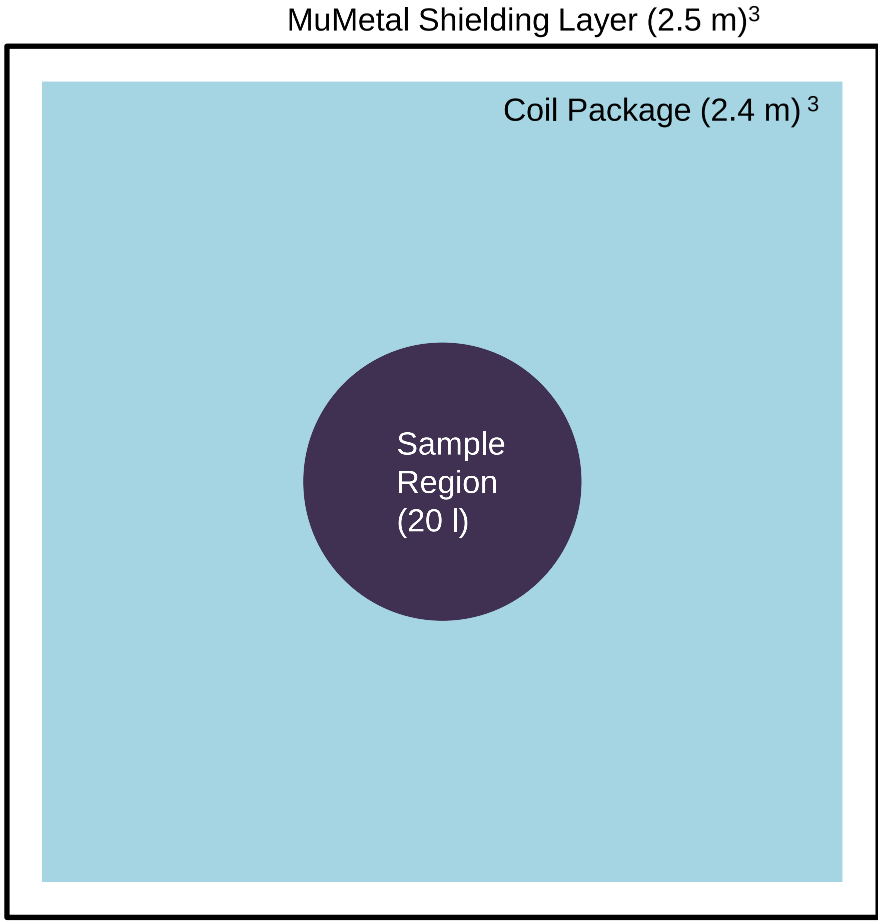
Everybody needs more JD Jackson in their life

Fields via Spherical Harmonics

Vm is defined on a closed volume

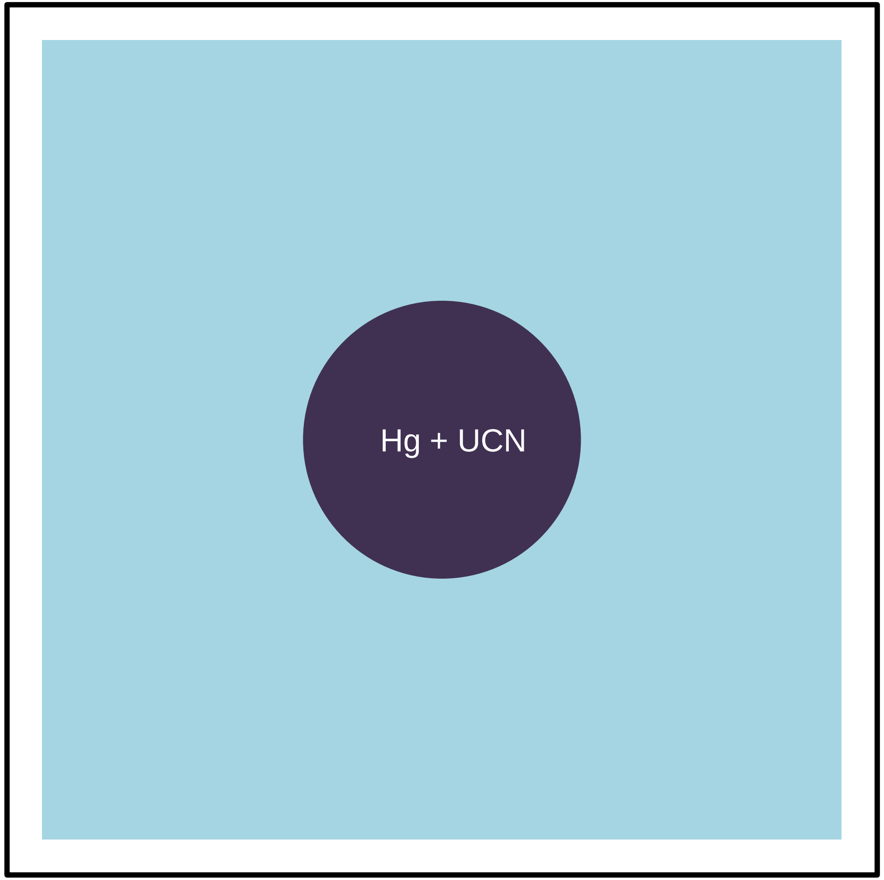
n.b. Ψ → Vm

- Wires along Vm contours approximate Vm within the full volume
- We're over-solving the problem
- We need holes in the coil surface for
  - Guides
  - Vacuum
  - HV
  - Optics
  - Etc…



We want Vm at a distance

- We apply current just inside the MSR
- We need a uniform field across the storage cells
- Gas species sample the full volume during storage.



Tx: Calculate, invert transfer function

- Given a full basis set of Vm: $\Sigma_{m,\ell}$
- Simulate coil response for each Σ
- Decompose coil response across sample region to $\Sigma_{m,\ell}$
- Orthogonalize response matrix, hope it isn't singular
- Generate a linear sum of Σ's that yield any desired $\Sigma_{m,\ell}$

$$\Sigma_{l,m} = C_{l,m}(\phi) r^l P_l^{|m|}(\cos\theta), \tag{4}$$

with

$$C_{l,m}(\phi) = \frac{(l-1)!(-2)^{|m|}}{(l+|m|)!}\cos(m\phi) \quad \text{for} \quad m \geqslant 0,$$
$$C_{l,m}(\phi) = \frac{(l-1)!(-2)^{|m|}}{(l+|m|)!}\sin(|m|\phi) \quad \text{for} \quad m < 0. \tag{5}$$

Finally, the modes are obtained by calculating the gradient of the magnetic potential:

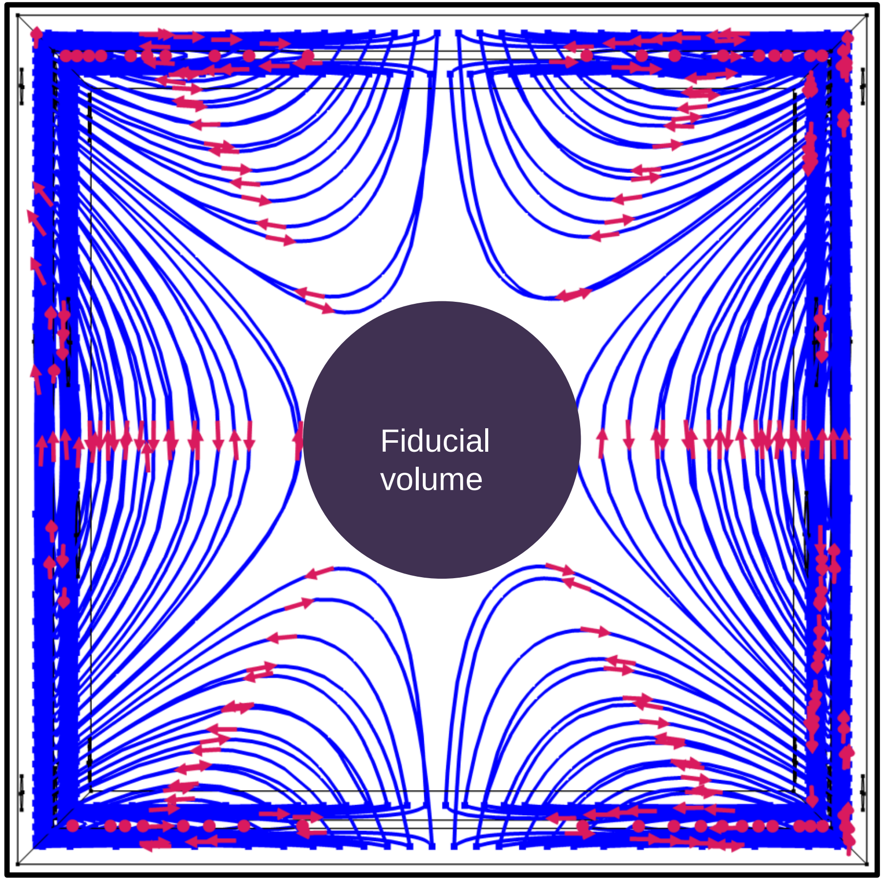
$$\Pi_{x,l,m} = \partial_x \Sigma_{l+1,m},\ \Pi_{y,l,m} = \partial_y \Sigma_{l+1,m},\ \Pi_{z,l,m} = \partial_z \Sigma_{l+1,m}. \tag{6}$$

TABLE II. The basis of harmonic polynomials sorted by degree.

l	m	Π_x	Π_y	Π_z
0	−1	0	1	0
0	0	0	0	1
0	1	1	0	0
1	−2	y	x	0
1	−1	0	z	y
1	0	$-\frac{1}{2}x$	$-\frac{1}{2}y$	z
1	1	z	0	x
1	2	x	$-y$	0
2	−3	$2xy$	$x^2 - y^2$	0
2	−2	$2yz$	$2xz$	$2xy$
2	−1	$-\frac{1}{2}xy$	$-\frac{1}{4}(x^2 + 3y^2 - 4z^2)$	$2yz$
2	0	$-xz$	$-yz$	$z^2 - \frac{1}{2}(x^2 + y^2)$
2	1	$-\frac{1}{4}(3x^2 + y^2 - 4z^2)$	$-\frac{1}{2}xy$	$2xz$
2	2	$2xz$	$-2yz$	$x^2 - y^2$
2	3	$x^2 - y^2$	$-2xy$	0
3	−4	$3x^2y - y^3$	$x^3 - 3xy^2$	0
3	−3	$6xyz$	$3(x^2z - y^2z)$	$3x^2y - y^3$
3	−2	$-\frac{1}{2}(3x^2y + y^3 - 6yz^2)$	$-\frac{1}{2}(x^3 + 3xy^2 - 6xz^2)$	$6xyz$
3	−1	$-\frac{3}{2}xyz$	$-\frac{1}{4}(3x^2z + 9y^2z - 4z^3)$	$3yz^2 - \frac{3}{4}(x^2y + y^3)$
3	0	$\frac{3}{8}(x^3 + xy^2 - 4xz^2)$	$\frac{3}{8}(x^2y + y^3 - 4yz^2)$	$z^3 - \frac{3}{2}z(x^2 + y^2)$
3	1	$-\frac{1}{4}(9x^2z + 3y^2z - 4z^3)$	$-\frac{3}{2}xyz$	$3xz^2 - \frac{3}{4}(x^3 + xy^2)$
3	2	$-x^3 + 3xz^2$	$-3yz^2 + y^3$	$3(x^2z - y^2z)$
3	3	$3(x^2z - y^2z)$	$-6xyz$	$x^3 - 3xy^2$
3	4	$x^3 - 3xy^2$	$-3x^2y + y^3$	0

Rx:

1. Solve for idealized Vm
2. Extract windings from #1
3. Add MuMetal, energize windings
4. Solve for Vm across fiducial volume
5. Decompose #4 into $\Sigma_{m,\ell}$
    - Surface or Volume integral?
    - Is there a difference in theory?
    - What about FE considerations?



Rx: CoilSolver

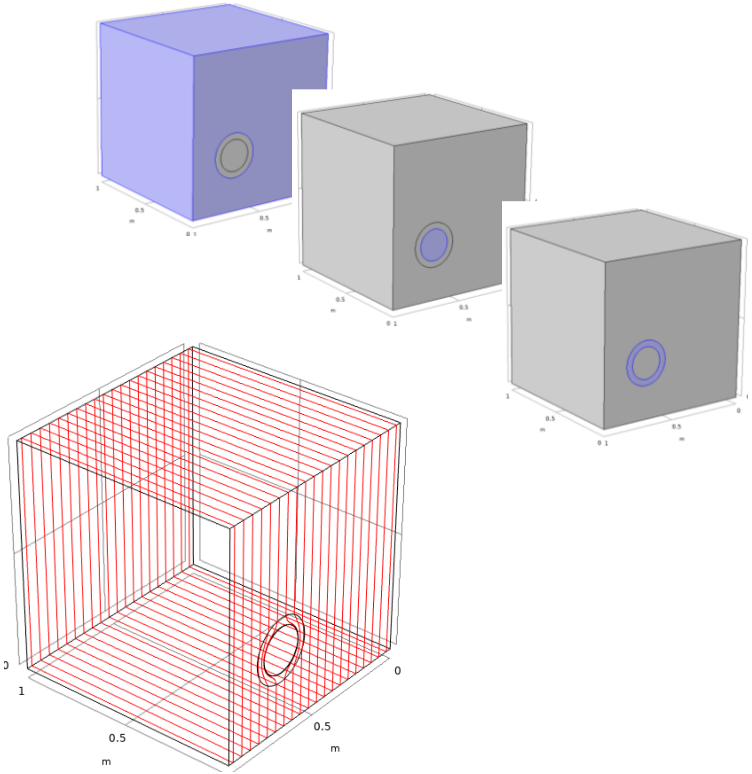
1. Build COMSOL model with geometry, physics, etc.
2. Point MATLAB at model, wait.
3. Specify a field. Get back its harmonic expansion.

GitHub:

https://github.com/MengerSponge/CoilSolver/

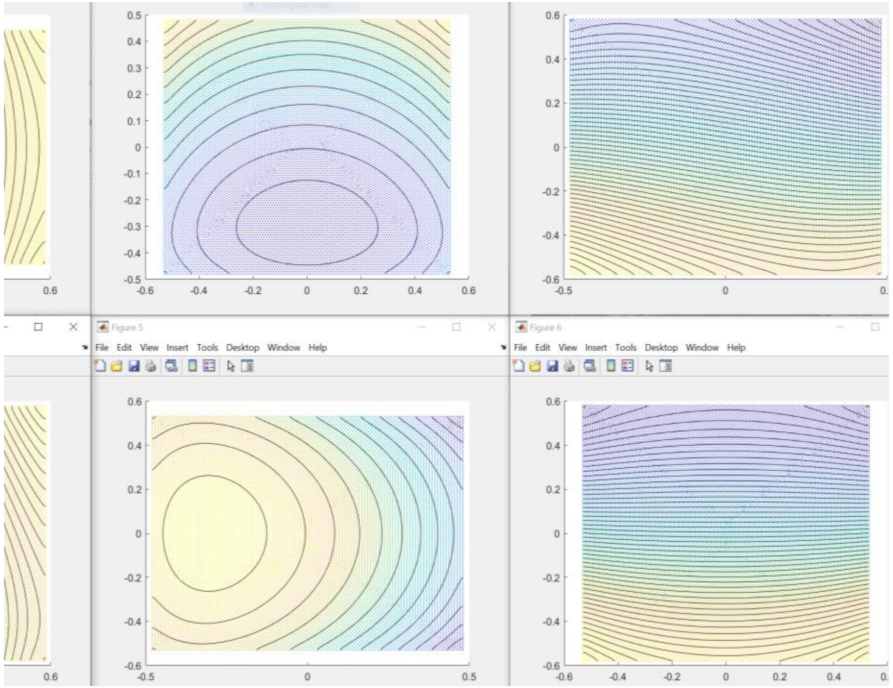
Step 1: Penetrations

1. Set surface to $\Sigma_{m,\ell}$
2. Set hole Vm to $\langle\Sigma_{m,\ell}\rangle_{hole}$
3. Allow annulus to float



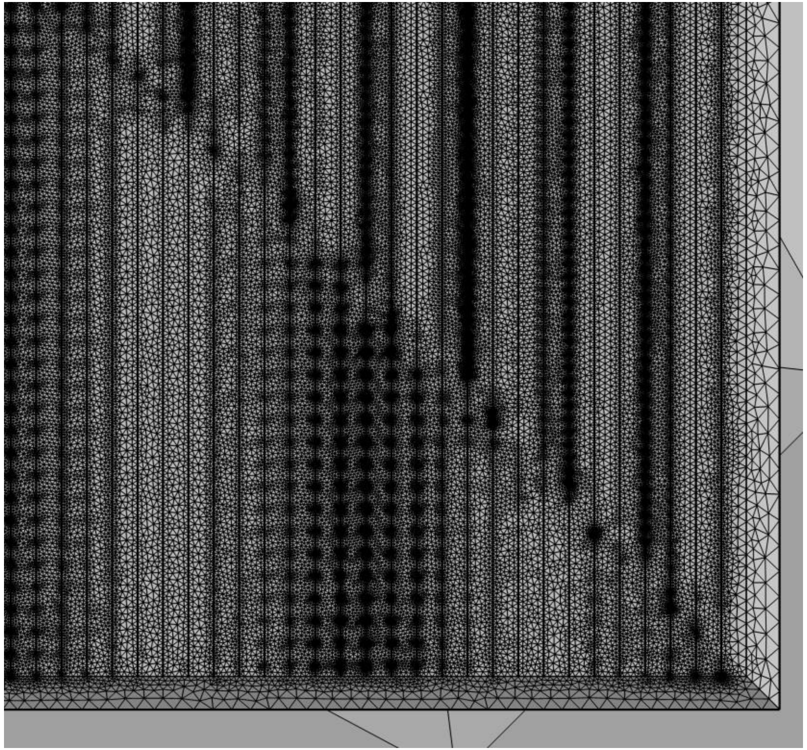
Step 2: Getting Contours

1. Pick a set of planar faces
2. Transform each one to 2D
3. Find contour lines across mesh
4. Correct direction of contour lines
   - Line collections in 2D are ready to build
   - Line collections need some processing to model robustly



Step 3: Mesh Wires

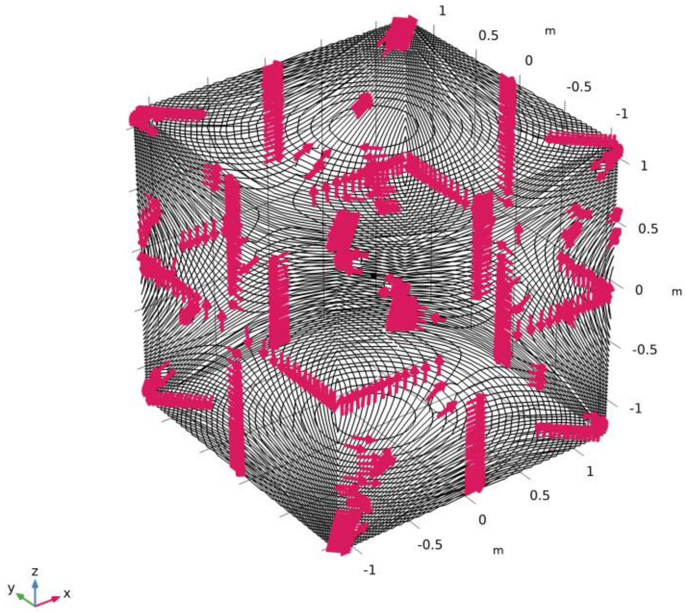
1. Points from plane contour are irregular
2. Irregular points lead to poorly defined interpolation curves
3. Resample each contour
4. Need finer resolution near penetration



Performance

1. 20-35 minutes per $\Sigma_{m,\ell}$
2. AMD Ryzen 2 Threadripper 2950X, 16-core@3.5GHz (4.4GHz Turbo)
3. 35 GB per $\Sigma_{m,l}$ (128 GB RAM DDR4-2666MHz available)
4. High order ℓ occasionally need some hand-holding to mesh and solve

Automated ℓ=2 m=0 coils with 12 penetrations



Open Questions

1. If linearizing low order harmonics excites high order ones, is there a preferred order we can use as a “dump stat”?
2. Current density?
   - Higher order harmonic corrections are smaller, so can we adapt the number of contour levels as a function of ℓ?
3. What if we restrict coils to certain faces?
4. How high does ℓ need to go? (ℓ=2?)

Conclusions

1. Able to generate manufacturable windings on any flat-faced closed surface.

2. Constant Vm patches yield reasonable penetrations

3. A method that relies on geometric nonlinearities isn't a great candidate for linear transfer functions

4. Is there a more clever way to iterate/cancel harmonic contributions?

This work is supported by Los Alamos National Laboratory LDRD and the National Science Foundation, grants PHY-1828512 and PHY-1614545

Thank you! Questions?

ausreid@iu.edu

INDIANA UNIVERSITY BLOOMINGTON

FULFILLING *the* PROMISE